Creating Data Sets from Statistical Measures

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Creating Data Sets from Statistical Measures* Illustration: This Illustration's student dialogue shows the conversation among three students who are asked to generate a set of 8 numbers that fit a given mean, median, mode and range. By using the meaning of the different statistics and working backwards, they are able to generate a data set and are left wondering if other data sets might also have met the problem's constraints.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 1: Make sense of problems and persevere in solving them.
MP 6: Attend to precision.
MP 7: Look for and make use of structure.

Target Grade Level: Grades 6–7

Target Content Domain: Statistics and Probability

Highlighted Standard(s) for Mathematical Content

6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
6.SP.B.5c	Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Math Topic Keywords: statistics, mean, median, mode, range, data sets

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Creating Data Sets from Statistical Measures

Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

Make up a set of eight numbers that simultaneously satisfy these constraints:
Mean: 10
Median: 9
Mode: 7
Range: 15

Task Source: Adapted from Falk, R. (1993). *Understanding Probability and Statistics: A Book of Problems*. Wellesley, MA: A.K. Peters.

Student Dialogue

Suggested Use
The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

Students have already learned how to calculate the mean, median, mode, and range of a data set. They are now working backwards to create data sets that fit a given set of statistics.

(1) Sam: Did we learn how to do this? There's no formula for this kind of problem, is there?

(2) Dana: No, we just need to find something we *can* do and see where it leads.

(3) Sam: Well, ok, it's gotta have a range of 15. So, let's just make the smallest number 1 and the largest 15 and see where that gets us.

(4) Dana: Well, 1 and *sixteen*. Or *zero* and 15. We want the *range* to be 15. That's the difference between smallest and largest, right?

(5) Sam: Oops, you're right. With 1 and 15 the, difference is only 14. We need a difference of 15. So, yeah, let's use 1 and 16. But that doesn't help us figure out any of the numbers in between.

(6) Anita: Well, it has a mode of 7, so there have to be more 7s than any other number. Doesn't have to be a lot of 7s, though, if all the other numbers are different. Two 7s would be enough. What's the point of this *mode* thing anyway? It seems like the mode is somewhat *meaningless* in a data set like this with only eight data points!

(7) Sam: Well, it's not meaningless in this puzzle, because it is one of our given constraints! Ok, so we have 1, 7, 7, 16 so far. The range is 15; the mode is 7. What else do we need?

(8) Dana: Well, right now the middle number—well, there isn't really a middle number, but as middle as we can get with just four numbers—is 7. We need the median to be 9. To make that middle number 9, we…

(9) Sam: …we could write 1, 7, 9, 7, 16. Ha!

(10) Dana: Very funny, Sam. You got the 9 into the middle. But seriously, we do need to put in enough numbers to get that 9 in the middle, even when they're in order, so that it will be the median. How about 1, 7, 7, 9, another number, another number, 16?

(11) Sam: Uh oh! That's seven numbers, with the 9 in the middle. If we put in an eighth number, there won't *be* a middle number.

(12) Dana: That's ok. *Between* the middle two numbers is ok, too. So, if the middle two numbers are both 9, then the median is 9. In fact, we could even make the middle two numbers 8 and 10, or even 7 and 11, because the average of those middle numbers stays 9.

(13) Anita: But then it gets so complicated. Let's just put another 9 in the middle of your list and work from there: 1, 7, 7, 9, 9, another number, another number, 16. We've nailed everything except the mean.

(14) Sam: So, we now have 1, 7, 7, 9, 9, x, y, 16, and we're trying to choose x and y so that we get a mean of 10. Right? Are we sure that's even *possible*?

(15) Dana: Oooh! Good question, Sam! To get a mean, we add all the values and divide by, um, 8 in this case, because we're using eight numbers. If we divide by 8 and the quotient is 10, the sum has to be 80. Uh oh!

(16) Anita: And we have 49 so far, so we need another 31.

[Students all work silently for a few minutes, writing numbers on their papers.]

(17) Sam: Wait!!! We *can* do it! We *can* pick two numbers between 9 and 16 that work! But just barely. I wonder if we could have solved this problem if we had started with 0 and 15. Or 7 and 22.

Teacher Reflection Questions

Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?

2. Oops! The students clearly demonstrate that they understand the meaning of each of the four given statistical measures, that they can calculate them correctly, and that they understand the implications for a data set well enough to know, for each measure, how to build or adjust a data set to accord with that measure. Yet, they have made some errors in their work. What are these errors and/or where did they slip up first? Other than "always remember to check your work," what idea or awareness might have helped them avoid the errors?

3. What adjustments *can* the students make to get a data set that fits the required constraints?

4. List some differences between this problem and a problem that starts with the data set and asks students to compute mean, median, mode, and range.

5. À la mode: Anita (line 6) asks, "What's the point of this *mode* thing anyway? It seems like the mode is somewhat *meaningless* in a data set like this with only eight data points!"
   A. Eight data points is too small a set for any measure to be very meaningful, but Anita's right: mode is (usually) especially meaningless in a small data set. Why?
   B. In a data set of exactly eight pieces of information, are there *any* circumstances in which mode might be the measure of choice?
   C. Under what circumstances *is* mode a useful measure or even the measure of choice? Under what circumstances would it appear *not* to be a useful measure?

6. Prove that there is not a set of 8 *positive integers* that simultaneously satisfy these constraints:
   Mean: 9
   Median: 10
   Mode: 7
   Range: 15

7. After line 14, what is a different way the students could think about how to find data points that satisfy the constraint of a mean of 10?

8. What tools would you provide to students working on this task, and why?

9. What are vocabulary demands you anticipate for your students if you provide them with this task?

Mathematical Overview

Suggested Use
The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	Evidence
 Make sense of problems and persevere in solving them.	In lines 1 and 2, Sam and Dana have not yet used any particular mathematical idea or specific characteristic of the problem—they are not yet actually solving—but they are already mentally scanning what they know and suggesting an entry point and a potential strategy for making further sense of the problem. This is the heart of MP 1. Furthermore, Sam, Dana, and Anita are certainly engaged in "making sense" of the problem at several levels, not the least of which is Anita's question (line 6) about the point of the mode statistic (though the students never actually tackle that question, and ultimately lose track of the mode altogether). Furthermore, throughout the dialogue the students work to adjust the statement of what they know about the set of numbers based on incorporation of additional constraints—they are constantly monitoring their progress and analyzing the reasonableness of their results.
 Attend to precision.	One characteristic of MP 6 is constantly going back to the definitions and constraints of a particular task. In this dialogue, for example, the students revisit what is meant by the "middle" or median value (lines 10–13). And earlier in this dialogue, the three students are careful to be explicit about what they mean by range and by mode to help make sure the different group members are talking about the same thing (lines 4–6).
 Look for and make use of structure.	These students demonstrate their engagement in MP 7 when they attend to how different data points will influence the mean, median, mode, and range of the entire data set. The students are able to anticipate some of the effects on these measures of adding particular data points before even calculating (lines 11–12). The students are also able to pick a smaller section of the data set to focus on (e.g., lines 7, 9, 10, 12–14) depending on the statistic they are working on. That is to say, they understand something about the structure of the data set and see how this "complicated thing" is "composed of several objects."

Creating Data Sets from Statistical Measures

Commentary on the Mathematics

Notes about this task

- Mode does not appear in the Common Core State Standards. It is included in this task as one constraint to help make the task promote student engagement in the mathematical practices and to help illustrate why mode is often not a particularly useful statistical measure. The statistical ideas from the task that are most relevant for students and which are highlighted in the CCSS are mean, median, and range.
- The problem posed here is about the mathematics behind the statistics, not an application of statistics; it is about understanding the mathematical interaction of four statistical measures. Computing those measures on a given data set is a straightforward problem with a single right answer. By contrast, constructing a data set from the measures requires attention to the nature and interaction of the measures, and leaves open the question as to how many correct answers there are (and what additional constraints, like the use of integer values only, might affect that number). Though this problem is about statistical measures, the contextualizing and decontextualizing that is so typical of statistical reasoning is not needed here. No context is assumed or needed, and we would not expect to see characteristics of MP 2 in students' reasoning about this problem.

When are MP 2 and MP 3 involved in work on statistical questions?
The problem posed here is about the mathematics *behind* the statistics, not an application of statistics; it is about understanding the mathematical interaction of four statistical measures. Computing those measures on a given data set is a straightforward problem with a single right answer. By contrast, constructing a data set from the measures requires attention to the nature and interaction of the measures, and leaves open the question as to how many correct answers there are (and what additional constraints, like the use of integer values only, might affect that number). Though this problem is *about* statistical measures, the contextualizing and decontextualizing that are so typical of statistical reasoning are not needed here. No context is assumed or needed, and we would not expect to see characteristics of MP 2 in students' reasoning about this problem.

This problem, therefore, is not typical of most statistical problems. The mathematics *behind* statistics is "pure mathematical" reasoning that verifies the various techniques of statistics and sets the parameters for where those techniques can be logically applied. The *purpose* and *application* of statistics, though, sits right at the interface between mathematics and the world of data—including data from the worlds of physical, biological, social, and other phenomena, and the worlds of mathematical processes. Statistics is, by nature, mathematics applied to a context, a place where we *must* decontextualize (turning real phenomena into numbers) and recontextualize (interpreting processed versions of those numbers back into descriptions of the real phenomena) regularly. Sensible applications of statistical reasoning require, at some point, the ways of thinking described in MP 2. To use statistical reasoning sensibly, we must understand not only its techniques and the mathematical constraints that govern the use of those techniques, but we must also "understand the data" and the context. That latter understanding is highly discipline specific and often a bit squishy to define. We must know when to "clean up" a data set by treating serious outliers as likely errors—noise that is irrelevant to or may even distort the answer to the question we're asking—and throwing them out. We must also know when *not* to modify the data set, to avoid the risk of forcing the data to accord with results we expect or want. Those decisions are not purely mathematical, though even they are sometimes aided by mathematical analysis.

It's also tempting to think that the discussion, especially because of its coherent give and take, illustrates MP 3's "constructing viable arguments and critiquing the reasoning of others." But if all intelligent, reasoned discussion is treated as MP 3, the special meaning of discussions that involve the construction and articulation of a logical *sequence of steps* can get lost. Of course, what's important is not whether some classroom interaction—this fictional one or some real one in your own class—gets "credit" for MP 3 in particular. What's important is that intelligent, reasoned discussion in your classroom include, over time—that is, not necessarily all in the same episode—the *variety* of communication suggested by the MPs: the back and forth of sense-making that is part of MP 1; the attempt to clarify and express more precisely that is part of MP 6; and the structured "viable argument," often as a part of articulating a process, an algorithm, or a proof, as well as the challenging each other's reasoning, that are part of MP 3.

What about *mode*?
In this dialogue, the question of the usefulness of mode is raised by one of the students (see Teacher Reflection Question 5). In the context of this mathematics task and dialogue, mode provides an interesting constraint for the problem, allowing for engagement in the MPs by the students. However, in practical use, mode is often not a useful statistical idea for small data sets. It is worth noting that mode does not appear in the *Common Core State Standards* at all. It is included in this task to help make the task one that will promote mathematical practices, but the statistical ideas from the task that are most relevant for students and which are highlighted in the CCSS are mean, median, and range.

Evidence of Content Standards
The CCSS content standards identified for this Illustration are 6.SP.A.3 and 6.SP.B.5c. These content standards emphasize the need for students to come to understand different measures of center and variation and to be able to determine these measures' correspondence to sets of data. The opportunities provided by a task such as this one—where students must work backwards from the mean, median, mode, and range, and through their explorations see how changes to the numbers in the data set impact these measures—allow students to deepen their understanding of these statistical measures.

Note: For additional information about statistics and probability in the *Common Core State Standards*, see the draft progressions documents for statistics and probability at http://ime.math.arizona.edu/progressions/

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Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. At the end of the dialogue, what numbers do Sam, Anita, and Dana have in their proposed data set? Do you agree that this set of numbers will work? If so, how? If not, why not?

2. What do the students say, in what lines, about what mean, median, mode, and range mean? How does sharing this information with each other help them work on the task?

3. How many of the eight numbers in the set did the students need to identify or check to know if the range could be correct? To know if the mode could be correct? To know if the median could be correct? To know if the mean could be correct?

4. In line 16, what does Anita mean by “we need another 31?”

5. In the mathematics task, no context was given. Assuming the context below, what information does a mean of 10, median of 9, mode of 7, and range of 15 give you?

Related Mathematics Tasks

1. Make up an additional set of eight numbers that also simultaneously satisfy these constraints:
   Mean: 10
   Median: 9
   Mode: 7
   Range: 15

2. A particular school has measured the height, to the nearest half-inch, of all the boys in one seventh-grade class and one eighth-grade class. (A) Which measure—median or range—would you expect to change more from one grade to the next? (B) Which measure—mode or mean—is likely to be more informative about the height of a student selected at random from one of the grades?

3. Make up a set of eight numbers that simultaneously satisfy these constraints. Can they all be positive integers?
   Mean: 10
   Median: 10
   Mode: 7
   Range: 15

4. Make up a set of eight numbers that simultaneously satisfy these constraints. Hint: They *cannot* all be positive integers.
   Mean: 9
   Median: 10
   Mode: 7
   Range: 15

Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?

   Refer to the Mathematical Overview for notes related to this question.

2. Oops! The students clearly demonstrate that they understand the meaning of each of the four given statistical measures, that they can calculate them correctly, and that they understand the implications for a data set well enough to know, for each measure, how to build or adjust a data set to accord with that measure. Yet, they have made some errors in their work. What are these errors and/or where did they slip up first? Other than "always remember to check your work," what idea or awareness might have helped them avoid the errors?

   The introduction of two 9s (line 13) creates a bimodal data set. The set no longer satisfies the requirement that *the* mode is 7. The students solve each sub-problem—first the range, then the mode, then the median—without being fully attentive to the fact that the four measures interact, so changes in the data set, even ones that on the surface appear to involve only the measure that they are currently working on, could be expected to affect other measures. We don't see how they solve the final sub-problem—getting a mean of 10—but Sam (line 17) says that they can "just barely" do it. So far, they have chosen only whole numbers for their data set; they may be about to do more damage to the mode, because the only two whole numbers that will give them the mean they want at this point would make the data set tri-modal.

3. What adjustments *can* the students make to get a data set that fits the required constraints?

   Dana almost gets there (line 12) with the suggestion of 7 and 11 as the middle numbers. If the students had chosen 7, 11 instead of 9, 9, they would have generated 1, 7, 7, 7, 11, another number, another number, 16. The mode is now firmly at 7 no matter what two values they choose for "another number."

4. List some differences between this problem and a problem that starts with the data set and asks students to compute mean, median, mode, and range.

   To do any problem that involves these measures, one must know how to compute them. A problem that starts with the data set effectively asks for nothing more than performing the correct computation. This task is "backwards"—students start with the measures and must construct a data set that accords with them—and so students cannot just follow a known procedure. One benefit is that they must analyze the definitions and see how to achieve them. They must also consider the ways the definitions interact, something that the students in this dialogue do not fully manage.

5. À la mode: Anita (line 6) asks, "What's the point of this *mode* thing anyway? It seems like the mode is somewhat *meaningless* in a data set like this with only eight data points!"
   A. Eight data points is too small a set for any measure to be very meaningful, but Anita's right: mode is (usually) especially meaningless in a small data set. Why?
   B. In a data set of exactly eight pieces of information, are there *any* circumstances in which mode might be the measure of choice?
   C. Under what circumstances *is* mode a useful measure or even the measure of choice? Under what circumstances would it appear *not* to be a useful measure?

It is possible to cook up special circumstances in which any of the measures is unstable and could change a lot depending on a change in just one data point. But Anita's point about mode in data sets like this one is quite right. Here's why:

To compute the mode in a data set, we treat the data set as having categories and we count the elements in each category. Mode is almost always unstable if data set is relatively small and the data are spread over many categories. Consider a data set of the ages of 25 students in one classroom, with the ages measured to the day. We might well find no mode at all: 25 different ages. One coincident birthday would be enough to create a mode, but the mode may tell nothing of statistical interest about the ages of students at this grade level or even in this class. Some other class performing the same experiment could have a very different mode. The measure can be found but has no meaning. If, instead, age is measured to the nearest month—only 12 categories instead of 365—a pileup of one particular value could have greater meaning because movement of a single data point would have less effect.

Votes are generally settled by mode. Especially if there are not many categories, mode can have significance. "The store has only strawberry and vanilla. Which flavor should we buy?" Even in just a tiny group, unless there are special circumstances (e.g., someone's allergy), mode is probably the measure of choice.

6. Prove that there is not a set of 8 *positive integers* that simultaneously satisfy these constraints:
   Mean: 9
   Median: 10
   Mode: 7
   Range: 15

   The proof requires some work. The median of an even number of points means that 10 is the mean of the middle two points. They could both be 10 or they could be, say, 9 and 11, or 8 and 12, and so on.

Case 1: If the middle two numbers are 10 and 10, the range would require three 7s to make the mode 7. We now have 7, 7, 7, 10, 10. To keep 10, 10 in the middle, the remaining three points must be greater than 10 (and not equal), so we have 7, 7, 7, 10, 10, x, y, z. The sum of the five known numbers is 41, so the $x+y+z$ must be 31 to achieve a mean of 9 (which requires a total sum of 72). To avoid changing the mode of 7, the minimum integer values of x, y, and z would be 11, 11, and 12, with a sum greater than 31. So we cannot even achieve the correct mean (let alone the correct range) with integers.

Case 2: If we choose 9 and 11 as the "middle two" to make the median 10, then the mode can be satisfied with two 7s (though more are allowable). Assuming only two 7s and preserving 9 and 11 as the "middle two" requires two points greater than 11, so we now have 7, 7, 9, 11, x, y. The remaining two points must be on opposite sides of the 9, 11 center: w, 7, 7, 9, 11, x, y, z (with w on either side of the two 7s). The total of the known numbers is 34, with 38 to go. If x, y, and z are integers and all different (so that they do not void the mode of 7), they must be 12, 13, and 14, totaling 39, requiring w to be –1 to reduce the total to 38. That satisfies all requirements including range, but then, not all numbers are positive. To keep w positive, x, y, and z could not be as large as 12, 13, and 14, but then not all numbers are integers.

Case 3: If the middle two are 8 and 12 or 7 and 13 instead of 9 and 11, then x, y, and z are forced yet higher, so w remains negative.

7. After line 14, what is a different way the students could think about how to find data points that satisfy the constraint of a mean of 10?

Students could attend to how far each data point is from the mean of 10. For example, they might write down how far each current data point is from 10 as follows:
1: –9
7: –3
7: –3
9: –1
9: –1
x:
y:
16: +6

From this list, they would see that they currently have numbers that are a total of 17 less than the mean and one number that is 6 more than the mean. So, the remaining two numbers (x and y) together need to be 11 more than the mean of 10.

8. What tools would you provide to students working on this task, and why?

   One tool that might be helpful is a computer spreadsheet. Students could use it to efficiently try different number combinations and see the impact on the mean, median, mode, and range.

9. What are vocabulary demands you anticipate for your students if you provide them with this task?

   As students are working on a task like this, they will be engaging with the vocabulary related to important mathematical ideas for the statistical terms mean, median, mode, and range. Students' work on the problem will indicate what their sense of these concepts is and whether intervention around the meaning of the terms is needed. For students who are English learners in particular, other words in the problem set-up, including "simultaneously" and "constraints," may require clarification prior to beginning work on the task. "Constraints" is an important term throughout mathematics, and students will benefit from learning the role of constraints. Another set of terms that is important to attend to are words and phrases such as "number" and "positive integer." Note that in this problem statement, students are asked to find any eight numbers. So, for example, a correct response could involve decimals.

Possible Responses to Student Discussion Questions

1. At the end of the dialogue, what numbers do Sam, Anita, and Dana have in their proposed data set? Do you agree that this set of numbers will work? If so, how? If not, why not?

   At the end of the dialogue, the students are working with 1, 7, 7, 9, 9, 16, and two numbers between 9 and 16. This won't work because 7 is supposed to be the mode, but both 7 and 9 will be modes with this set of numbers.

2. What do the students say, in what lines, about what mean, median, mode, and range mean? How does sharing this information with each other help them work on the task?

   In line 4, Dana asks about range: "That's the difference between smallest and largest, right?"

   In line 6, Anita says, "It has a mode of 7, so there have to be more 7s than any other number. Doesn't have to be a lot of 7s, though, if all the other numbers are different. Two 7s would be enough."

   In line 15, Dana explains that "To get a mean, we add all the values and divide by, um, 8 in this case, because we're using eight numbers."

   In lines 8 through 12, Dana and Sam discuss what the median is. When Sam says (line 11), "If we put in an eighth number, there won't *be* a middle number," Dana replies, "*Between* the middle two numbers is ok, too."

Sharing their thinking about these different statistics helped the students decide what numbers to try in their set of eight numbers and to figure out what numbers would not work.

3. How many of the eight numbers in the set did the students need to identify or check to know if the range could be correct? To know if the mode could be correct? To know if the median could be correct? To know if the mean could be correct?

   Range: The lowest and highest numbers in the data set are all that matter.
   Median: The fourth and fifth highest numbers in the data set are all that matter.
   Mean: All eight numbers are needed to check the mean.
   Mode: All eight numbers are needed to check the mode if there are only two 7s for the mode (to ensure there are not two of any other number).

4. In line 16, what does Anita mean by "we need another 31?"

   The students have found six numbers so far that they think will work: 1, 7, 7, 9, 9, and 16. These six numbers add to 49. If they find two more numbers that add to 31, the total of all eight numbers will be 80 (49 + 31). And with a total of 80 for 8 numbers, the mean is 10 (which is the mean that the math task asks for). So, the students think they need two more numbers that add to 31 to complete the set of eight numbers.

   Anita might also be looking at "another 31" another way. Here is a list of how far away each number is from 10:
   1: –9
   7: –3
   7: –3
   9: –1
   9: –1
   x:
   y:
   16: +6

   From this list, Anita could realize that the numbers they have so far add up to being 11 less than the mean. The "31 more" that are needed are 10 for each of x and y plus 11 more than the mean to counteract the 11 less than the mean that already exist in the list of numbers.

5. In the mathematics task, no context was given. Assuming the context below, what information does a mean of 10, median of 9, mode of 7, and range of 15 give you?

   Context: Maria had never run a race before but decided to train for a race for eight weeks and see how far she could run. She recorded the total number of miles she ran each week for eight weeks. Each week she ran at least as many miles as the week before.
   Mean = 10 miles. She must have run 80 miles total over the 8 weeks.

Median = 9 miles. She was running less than or equal to 9 miles up through the fourth week and more than or equal to 9 miles starting with the fifth week.
Mode = 7. During at least two weeks she ran 7 miles total.
Range = 15. She was able to run 15 more miles in the last week than in her first week of training.

Possible Responses to Related Mathematics Tasks

1. Make up an additional set of eight numbers that also simultaneously satisfy these constraints:
   Mean: 10
   Median: 9
   Mode: 7
   Range: 15

   Here are a few examples of sets of numbers that satisfy the constraints:
   7, 7, 7, 9, 9, 9.5, 9.5, 22
   2, 7, 7, 8, 10, 13, 16, 17
   2, 7, 7, 7, 11, 13, 16, 17

2. A particular school has measured the height, to the nearest half-inch, of all the boys in one seventh-grade class and one eighth-grade class. (A) Which measure—median or range—would you expect to change more from one grade to the next? (B) Which measure—mode or mean—is likely to be more informative about the height of a student selected at random from one of the grades?

   A. The median will almost certainly change more. If all of the students—the shortest, the tallest, and those in between—grow about two inches between seventh and eighth grade, the range will stay pretty much the same even though all students are taller.

   B. If most boys in the class had exactly the same height, then mode would be the best predictor, but that would be a very strange circumstance! In normal circumstances, heights vary. Though it is likely for two or three boys to have the same height, the coincidence could be among the tallest boys or the shortest or anywhere in the middle. It is even possible for all of them to be different. In that case, there would be no mode at all. In any of those cases, mode would give no information about the height of the class. While the mean might not be the best way to predict the height of someone picked at random from the class—one very tall boy could change the mean without changing what you should predict about the class—it is certainly a better predictor than the mode.

3. Make up a set of eight numbers that simultaneously satisfy these constraints. Can they all be positive integers?
   Mean: 10
   Median: 10
   Mode: 7
   Range: 15

   Here are a few examples of sets of numbers that satisfy the constraints, two of which are all positive integers:
   1, 7, 7, 8, 12, 14, 15, 16
   1, 7, 7, 7, 13, 14, 15, 16
   2, 6, 7, 7, 13, 13.5, 14.5, 17

4. Make up a set of eight numbers that simultaneously satisfy these constraints. Hint: They *cannot* all be positive integers.
   Mean: 9
   Median: 10
   Mode: 7
   Range: 15

   Here are a few examples of sets of numbers that satisfy the constraints:
   –0.5, 7, 7, 9.5, 10.5, 11.5, 12.5, 14.5
   –1, 7, 7, 9.2, 10.8, 12, 13, 14
   –1.65, 7, 7, 7, 13, 13.15, 13.15, 13.35