Modeling Problem—Biking Home

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Modeling Problem—Biking Home* Illustration: This Illustration's student dialogue shows the conversation among three students who are asked to create a model showing where Tommy can be, given certain rate conditions, while still being able to make it home on his bike in one hour. As the students consider different cases they build geometric constructions to represent the area Tommy can travel in while still meeting the constraint of biking home in an hour.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 1: Make sense of problems and persevere in solving them.
MP 4: Model with mathematics.
MP 8: Look for and express regularity in repeated reasoning.

Target Grade Level: Grades 9–10

Target Content Domain: Modeling with Geometry (Geometry conceptual category)

Highlighted Standard(s) for Mathematical Content

G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Math Topic Keywords: circles, constraints, speed, distance, rate

This material is based on work supported by the National Science Foundation under Grant No. DRL-1119163. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

Tommy lives out in the country. A straight road passes his house, with lots of open grassy field on either side. He is allowed to ride his bike anywhere as long as he can get home in ONE hour or less from the time his parents call him on his cell phone. He can ride fast on the road (10 mph) and less fast in the grassy area (6 mph). Create and explain a model that shows where Tommy can be and still be able to get home within an hour.

Student Dialogue

Suggested Use
The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

Students in this Student Dialogue have experience with properties and area formulas of different shapes, including circles.

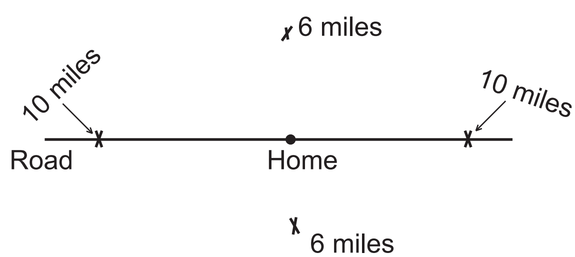
(1) Lee: There's a lot going on in this problem. Let's start with a picture. *[draws picture]*

(2) Chris: Sure, that looks right. But, now we need to figure out how far Tommy can bike on the road and in the grass in one hour so we'll know how far away he can be when his parents call.

(3) Lee: This is complicated. How do we know how much time he spent riding on each?

(4) Matei: Well, we don't actually care what he did on his way out. He could be dropped by helicopter. All we need to know is that his trip back, the fastest possible way, takes an hour or less. Let's keep it simple for now. Where could Tommy be if his trip home uses *only* the road and no grass? And where could he be if he bikes *only* on the grass to get home?

(5) Lee: That's easy enough! We know he can ride 10 miles in an hour on the road and 6 miles in an hour on the grass, so that's how far away he can go. *[marks two Xs on the road on either side of home and labels them 10 miles; and marks two Xs in the grass on either side of home and labels them each 6 miles]*



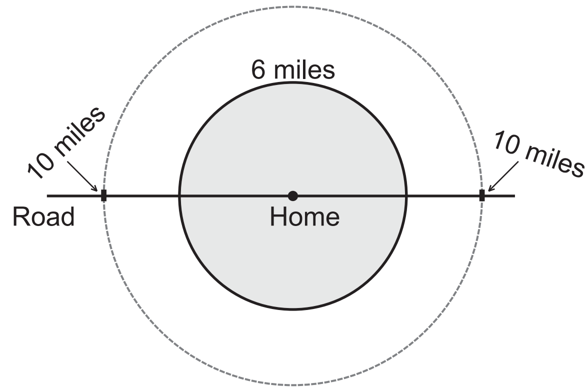
(6) Chris: Those aren't the only places he could go. He doesn't have to take the whole hour to get back, so he could be closer.

(7) Lee: Right, I was just trying to show the farthest he could be. He can be anywhere between these two points on the road *[points to the locations]* and anywhere between these two points on the grass.

(8) Chris: And that's not all.

(9) Matei: Oh yeah. He could go out 6 miles in any direction from his home. *[draws a circle with radius 6 with the Home as center]*

(10) Lee: OK, right. Let me redraw this a bit more accurately. *[Lee takes out a compass, measures a 10-cm radius, draws a light circle centered at home, marks two spots on the road on either side of home, and labels them 10 miles. He then draws another circle centered on home, this time measuring a 6-cm radius, shades it in lightly and labels it 6 miles]*

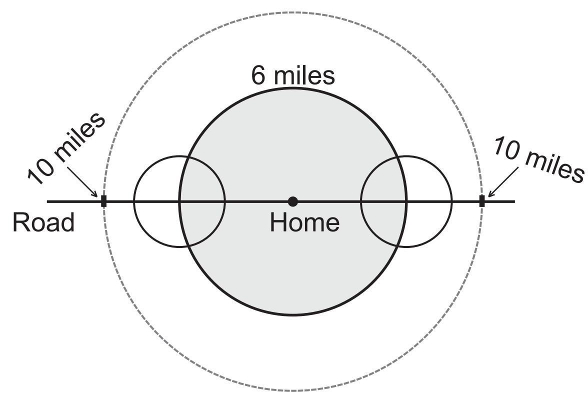


So is that our answer? A circle with radius 6 and a line along the road going 10 miles out in either direction?

(11) Chris: *[Thinks a bit.]* Well, no! Imagine he rides *along the road* the six miles to the edge of that circle. He'll have time to ride *off* the road, *beyond* the circle. But how much? Um, he rides 10 miles in an hour. So he rides 1 mile in a tenth of that time, um, 6 minutes, so it takes him 36 minutes to ride the 6 miles on the road. OK, from there, he has 24 minutes to ride out into the grass and still be no farther than an hour from home.

(12) Matei: Yeah, it seems like he should be able to.

(13) Lee: *[Lee takes out the compass, figures out how far Tommy could ride in 24 minutes, sets the compass, and draws two circles showing where Tommy could get to.]* Something like this?



(14) Chris: Well, yeah, but Tommy could ride almost all the way on the road and still get at least a little way onto the fields. There's got to be more.

(15) Lee: I guess we need to look at some examples. So, he could bike half an hour on the road *[traces finger from home to a point about halfway out to the 10 mile mark]* and then half an hour on the grass *[moves finger out into the grass a little ways]*. Or he could go 45 minutes on the road [*traces finger from home to a point about* $\frac{3}{4}$ *of the way out to the 10 mile mark]* and then 15 minutes on the grass *[moves finger out into the grass a little ways]*.

(16) Matei: I think we're going to need to draw circles at all these points you're trying, Lee. That would show how far he can get in the grass from that spot on the road. The circles will just get smaller the farther out on the road you go because he'll have less time in the grass.

(17) Lee: And all those circles together will be our answer!

(18) Chris: So, how much smaller will the circles get? Let's try a bunch of examples and see whether we can find a pattern we can use, or even an equation.

Teacher Reflection Questions

Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

2. What shapes might students predict for the area where Tommy can go and what would such shapes reveal about a student's thinking?

3. As students explore the shape of the area where Tommy can go, what hints or questions might you ask students and when or why would you intervene?

4. What productive mathematical ideas could students explore when working on this task, even though they may not be able to represent all points that are part of the solution?

5. Given the problem in the Student Dialogue, answer the following:
   A. If Tommy were to ride on the road for 1 mile before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   B. If Tommy were to ride on the road for 2 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   C. If Tommy were to ride on the road for 3 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   D. If Tommy were to ride on the road for d miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?

6. If you draw a picture (to scale) representing the area that Tommy can ride on the grass after he veers off the road after different distances, what do you notice about the picture? What conjecture could you make? Note that this is what the students in the Student Dialogue proposed to do in lines 16 and 18.

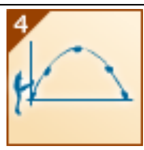
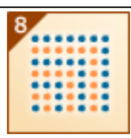
7. If you were to work on the problem in the Student Dialogue in the Cartesian plane, with home as the origin, what equation could you write for the circle describing how far Tommy can ride on the grass if he veers off the road after traveling it for d miles?

8. What tools would you give students to work on this task? Why?

Mathematical Overview

Suggested Use
The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	Evidence
 Make sense of problems and persevere in solving them.	In the Student Dialogue, we can see students persevering through an open-ended problem. They begin by “explaining to themselves the meaning of a problem” when reasoning that they should look at how far Tommy can ride in an hour if they want to see how far Tommy can be from the house and still make it back in time if his parents call (line 4). Students also look at “special cases… of the original problem” by considering the extreme case of Tommy driving only on the road or only on the grass (lines 5–10). After looking at those special cases, they can then begin to think about paths Tommy might take that involve riding on both road and grass.
 Model with mathematics.	Students are using geometric shapes and their properties to model the scenario and the area in which Tommy can bike. Students use a line segment to represent the road and a point to represent the house (line 1). Students also make use of the properties of a circle, specifically that it contains points in all directions that are a constant distance, called radius, from the center. A circle is used in Matei’s understanding of what it means to move in all directions (line 9).
 Look for and express regularity in repeated reasoning.	After looking at the special cases of Tommy riding only on the road or only on the grass, students begin to look at cases where Tommy rides on both. It is during this work that students begin to use repeated reasoning. After trying an example where Tommy rides 6 miles on the road and the rest on the grass (line 13), students recognize that this type of image could be repeated after Tommy rides some other distance down the road (lines 14–16). Even though students do not have a full solution by the end of the Student Dialogue, they realize that all those circles combined will give an answer (line 17) and are on their way to “express [the] regularity in [their] repeated reasoning.”

Commentary on the Mathematics

The task in the Student Dialogue can be approached using an experimental method as described in Teacher Reflection Question 6.

The key insight is that Tommy can ride along the road some distance between 0 and 10 miles, and then his path on the grass is constrained to a circle whose radius depends on where he is along the road. Though the students look in both directions from home, and in both directions from the road, the situation's symmetry means that they can analyze only, say, the upper right "quadrant" of this arrangement, and that will apply to all quadrants.

From line 15 on, the Student Dialogue moves in the direction of systematic experimentation with the ideas generated earlier: choose a distance along the road, figure out how much time that takes, figure out how far the remaining time allows one to go in the grass, and draw the circle. In line 11, Chris outlines the calculations step by step for a specific case, riding six miles along the road. He calculates in minutes, but that actually complicates the calculation. To ride m miles from home, Chris calculates it takes $\frac{m}{10}$ hours, leaving $\left(1-\frac{m}{10}\right)$ hours to ride on the grass.

Riding at 6 miles/hour for that amount of time lets Tommy go a $6\left(1-\frac{m}{10}\right)$-mile radius on the grass. Calculations get easier if we think of the drive of m miles as leaving $10-m=d$ miles to go and work the calculations from that. In that case, Tommy has a $\frac{6}{10}d=\frac{3}{5}d$ mile radius to go on the grass.

Students can draw circles by hand or set this up in *Geometer's Sketchpad* or whatever geometry software suits them.

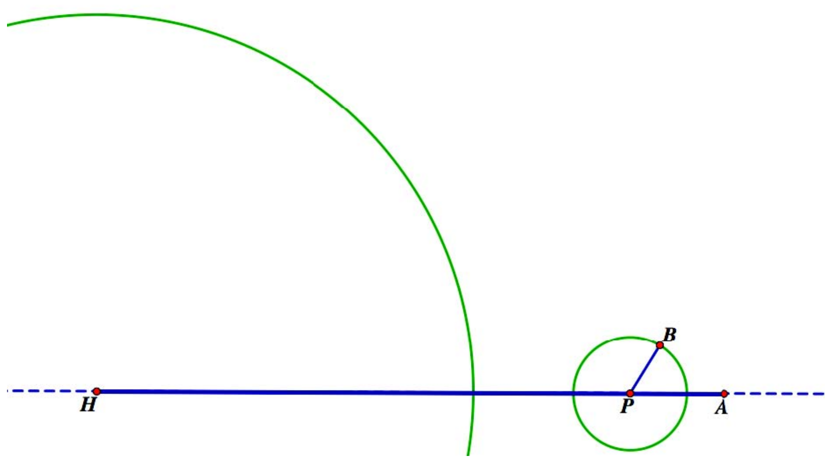


Figure A: The basic setup

Modeling Problem—Biking Home

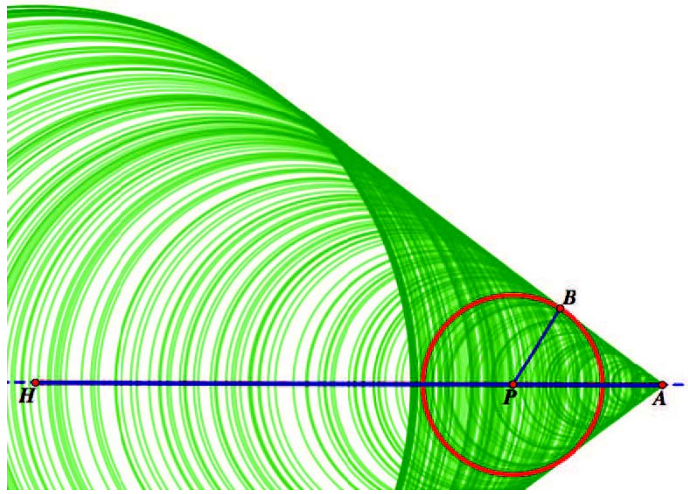


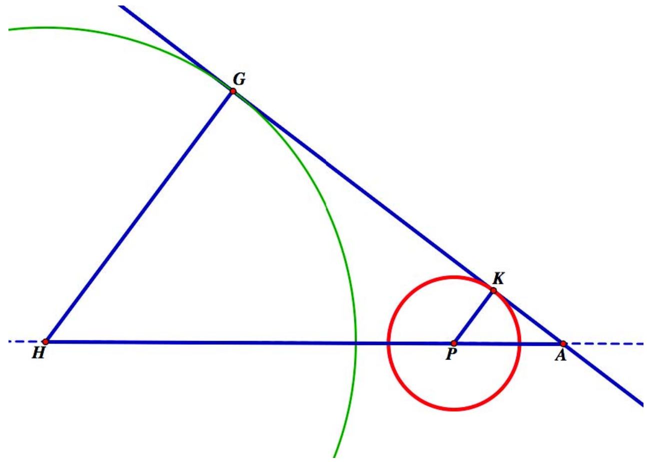
Figure B: A large number of circles, drawn according to the calculation

The circle around *H* is, as Lee drew it, radius 6; at a full 10 miles from home (the point *A* in Figure B), there is *no* extra time to ride on the grass, and the in-between circles all seem to be tangent to a single line through *A* and tangent to the original 6-mile-radius circle around home.

Why this shape?

Depending on their background knowledge, students may or may not reach this conjecture about the line to which these circles are tangent, and even if they do, they will not necessarily explore why this shape shows the boundaries of where Tommy can ride. If you are interested in exploring this question more, here is one way to think about it: The paths that allow Tommy to get home in time (using the notation of the constructions above) consist of $\overline{HP}$ followed by any straight line path from *P* to a point on or inside a circle of radius of $\frac{3}{5}PA$ around *P*. Let's call this "Circle P." We want to show that for any choice of *P*, these circles are tangent to the line from *A* to the big circle of radius 6 around *H*.

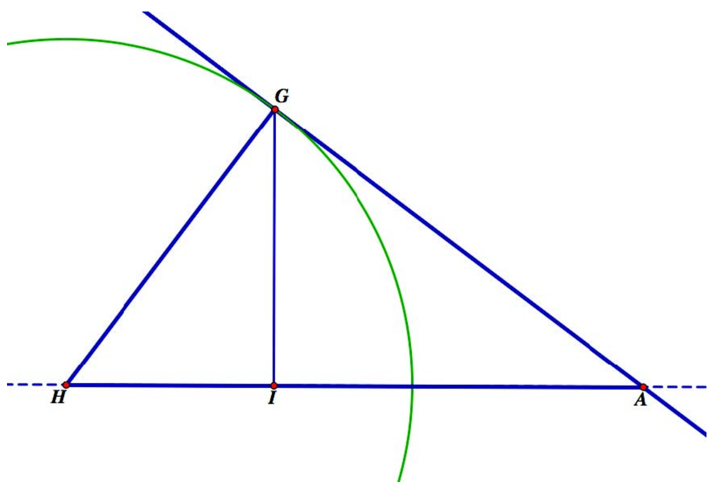
1. Draw the tangent. Label its intersection with Circle H *G*.
2. We know that $HG = 6$, $HA = 10$, and $\angle HGA$ is a right angle. That makes $AG = 8$ by the Pythagorean Theorem, and so the triangle is a 3-4-5 right triangle.
3. *Some* circle—let's call it *Mystery Circle*—around *P* is tangent to $\overleftrightarrow{AG}$, though we don't yet know whether it is the same circle we've called Circle P.



Is the *Mystery Circle* we see here the one we called Circle P?

4. $\triangle HGA$ and $\triangle PKA$ share $\angle A$ and have a right angle each, so they are similar.
5. Therefore, the ratios of the sides of $\triangle PKA$ is the same as the corresponding ratios in $\triangle HGA$. The radius PK of our *Mystery Circle* must therefore be $\frac{3}{5}PA$.
6. That's it! Circle P and *Mystery Circle* have the same center and radius, so they are the same circle. Since *Mystery Circle* was constructed to be tangent to $\overleftrightarrow{AG}$, we now know that Circle P—for all choices of *P*—is tangent to that line.

This situation can be analyzed further by looking at the similar triangles in the following figure:



Evidence of the Content Standards

In this Student Dialogue, students use geometric constructions (e.g., points, lines, circles) to model various scenarios (G.MG.A.3). Through their constructions, students begin to see what a solution to the task might look like.

Modeling Problem—Biking Home

Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. What are some of the strategies students in the dialogue use as they first start to tackle the problem?

2. If Tommy switches directions while on the grass, how does that impact how far he can go?

3. In line 11, Chris considers what happens if Tommy rides his bike on the road for 6 miles before riding on the grass. How far can he go on the grass from the point he veers off the road? How do you know this?

Related Mathematics Tasks

1. Given the problem in the Student Dialogue, answer the following:
   A. If Tommy were to ride on the road for 1 mile before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   B. If Tommy were to ride on the road for 2 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   C. If Tommy were to ride on the road for 3 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   D. If Tommy were to ride on the road for d miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?

2. In Student Discussion Question 3 and Related Mathematics Task 1, you found the distance that Tommy could ride on the grass if he veers off the road after riding on the road for 1, 2, 3, and 6 miles. Draw a picture to scale representing the area that Tommy can ride on the grass after he veers off the road. (Or, better yet, create this picture using geometry software.) What do you notice about your picture? What conjecture can you make?

Modeling Problem—Biking Home

Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

   Refer to the Mathematical Overview for notes related to this question.

2. What shapes might students predict for the area where Tommy can go and what would such shapes reveal about a student's thinking?

   Most students likely will not determine the full solution in regard to the final shape that represents all points where Tommy can go, as this requires some rather advanced mathematics (see Question 6). However, students can engage in some productive modeling and mathematical explorations as they determine what they do know about where Tommy can ride. During this exploration, students might predict various shapes for the area Tommy can go. By analyzing the students' answers, we can see what features of the problem they are considering.

   For example, if a student predicts a rectangle that is 10 miles wide and 6 miles long, we know the student understands that two quantities are at play but doesn't realize that every minute Tommy bikes on the road is one less minute he is riding on the grass.

   If students predict a rhombus shape (e.g., by connecting the four points that the students in the Student Dialogue found in line 5), then they are correctly considering two extreme cases (biking only on grass or biking only on the road) as represented by the vertices of the rhombus, and they seem to understand that every minute not spent biking on the road means biking on the grass; however, they are incorrectly assuming that biking on the grass is automatically done in a direction perpendicular to the road.

   If students predict an oval shape, they may understand that the area Tommy can travel could be represented as a series of circles each with a center somewhere along the road. The farther away from the house that Tommy begins to bike on the grass, the smaller the circle will be. Assuming that a series of such circles will form an oval is a reasonable guess, though ultimately incorrect as can be seen in Question 6.

3. As students explore the shape of the area where Tommy can go, what hints or questions might you ask students and when or why would you intervene?

    Depending on where students are in their understanding of the problem and how much they have worked individually or together to reason through a possible solution, some questions or prompts might be helpful. Possible questions and prompts include:

    - How fast can Tommy ride on the road? On the grass?
    - What is the farthest from home that Tommy can ride and still get back in an hour?
    - Draw 10 different points that show places Tommy can ride to and still get back in an hour.
    - If Tommy rides for $\frac{1}{4}$ hour on the road before switching to riding on the grass, how far will he ride on the road and how far will he ride on the grass? (This question may help prompt students' thinking about the different speeds Tommy can ride in the grass versus on the road and how that translates into distance traveled.)
    - Sketch the region in which Tommy can travel if he has veered off the road after 0 minutes? After $\frac{1}{4}$ hour? $\frac{1}{2}$ hour? $\frac{3}{4}$ hour? 1 hour? (This question may help students visualize the decreasing circles of area that Tommy can ride inside as he is farther away from the house.)
    - What is the radius of the circle Tommy can ride in on the grass if he veered off the road after a distance of d miles? What is the equation of that circle? (This question may help students begin moving to an algebraic equation that describes the scenario and may be graphed on the Cartesian plane. Such a direction may lead students to a solution similar to that found in Question 7.

4. What productive mathematical ideas could students explore when working on this task, even though they may not be able to represent all points that are part of the solution?

    This task allows students to explore what it means to answer an open-ended problem and what type of solution might satisfy such a problem. The question asks "Where can Tommy ride and still make sure he is able to get home in time?" which requires students to figure out how they might present a solution. Using a sketch? A graph on the coordinate plane? An equation?

    Also, students might explore different problem-solving strategies such as looking at extreme cases as done in the Student Dialogue. Looking at extreme cases and then several in-between examples can shed light on how the problem behaves and allow students to guess at what the solution might be.

    In terms of mathematical content, some students might use the constraints of the problem to generalize a formula for the radius of a circle where Tommy can ride on the grass based on when he gets off the road. They can also write equations for these circles and begin to explore how tangents of these circles form a portion of the boundary defining where Tommy can bike.

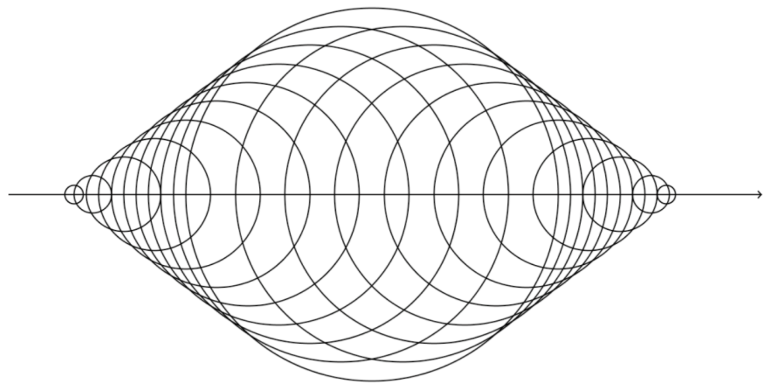
Modeling Problem—Biking Home

5. Given the problem in the Student Dialogue, answer the following:
   A. If Tommy were to ride on the road for 1 mile before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   B. If Tommy were to ride on the road for 2 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   C. If Tommy were to ride on the road for 3 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   D. If Tommy were to ride on the road for d miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?

   A. If Tommy rode on the road for 1 mile, that means he had been traveling for $\frac{1}{10}$ of an hour. Since he has $\left(1-\frac{1}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{1}{10}\right)\cdot 6$ or 5.4 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.
   B. If Tommy rode on the road for 2 miles, that means he had been traveling for $\frac{2}{10}$ of an hour. Since he has $\left(1-\frac{2}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{2}{10}\right)\cdot 6$ or 4.8 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.
   C. If Tommy rode on the road for 3 miles that means he had been traveling for $\frac{3}{10}$ of an hour. Since he has $\left(1-\frac{3}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{3}{10}\right)\cdot 6$ or 4.2 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.
   D. If Tommy rode on the road for d miles, that means he had been traveling for $\frac{d}{10}$ of an hour. Since he has $\left(1-\frac{d}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{d}{10}\right)\cdot 6$ miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.

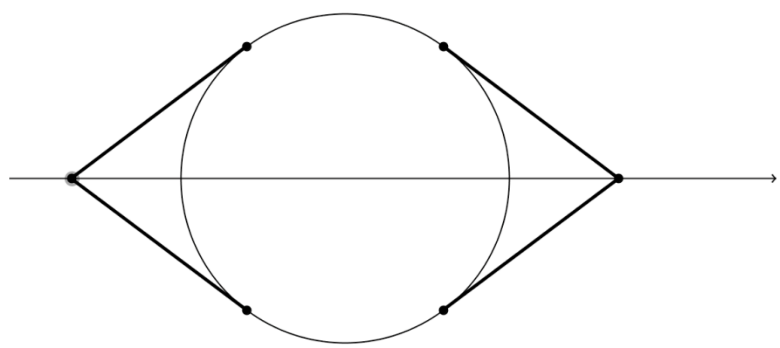
6. If you draw a picture (to scale) representing the area that Tommy can ride on the grass after he veers off the road after different distances, what do you notice about the picture? What conjecture could you make? Note that this is what the students in the Student Dialogue proposed to do in lines 16 and 18.

An image at scale of the circles representing the area Tommy can ride on the grass after he veers off the road can be seen below.



Looking at the image above, one might notice that the circles appear to form a straight line along the outer perimeter of the region that Tommy can ride and still be back home within an hour. This observation most likely would occur in circumstances where the image is very accurately drawn, such as one produced using geometry software.

One conjecture that can be made is that the final solution to the problem can be drawn as follows: (1) draw a 6-mile circle around home, (2) mark down the two points along the road that are 10 miles away from home, and (3) form line segments tangent to the circle and connecting the two points on the road. This conjecture of the final solution would look like the image below.



7. If you were to work on the problem in the Student Dialogue in the Cartesian plane, with home as the origin, what equation could you write for the circle describing how far Tommy can ride on the grass if he veers off the road after traveling it for *d* miles?

   If the house is represented by the origin on the coordinate plane and the road is represented by the *x*-axis, then the point that Tommy veers off the road and rides on the grass is given by $(d,0)$. It is important to note that at this point d can be a positive value if Tommy traveled to the right of his house or a negative value if Tommy traveled to the left of his house. If Tommy does not ride on the road at all, the equation of the circle where he can ride in the grass is: $x^2 + y^2 = 36$

   If Tommy rides 1 mile to the right on the road, the equation is as follows:
   $(x-1)^2 + y^2 = 29.16$ (see Teacher Reflection Question 5 for more about why 29.16)

   If Tommy rides 2 miles to the right on the road: $(x-2)^2 + y^2 = ((1-.2)\cdot 6)^2$

   If Tommy rides 3 miles to the right on the road: $(x-3)^2 + y^2 = \left(\left(1-\frac{3}{10}\right)\cdot 6\right)^2$

   If Tommy rides 4 miles to the right on the road: $(x-4)^2 + y^2 = \left(\left(1-\frac{4}{10}\right)\cdot 6\right)^2$

   A regular structure is emerging as the examples proceed. The equation for the circle where Tommy can ride if he rides *d* miles to the right on the road must be:
   $(x-d)^2 + y^2 = \left(\left(1-\frac{d}{10}\right)\cdot 6\right)^2$

   And looking at the symmetry of the context, we know that there is a duplicate set of circles if Tommy rides out to the left on the road.

8. What tools would you give students to work on this task? Why?

   A variety of tools could be used when working on the task, and it is important to consider the benefits and constraints of selecting different tools. Students can use paper and pencil to engage in some amount of sketching out of possible routes that Tommy can travel. As students proceed to considering circles of various sizes, it can become difficult to sketch these circles accurately enough to notice patterns that can emerge from the circles. So, giving students access to computers or calculators with graphing capabilities could benefit students after some initial sketching and exploration of the problem context. If access to geometry software is not possible, other possible tools that could support students' exploration include string, rulers, grid paper, and compasses.

Modeling Problem—Biking Home

Possible Responses to Student Discussion Questions

1. What are some of the strategies students in the dialogue use as they first start to tackle the problem?

   Students in this Student Dialogue use pictures to visualize the problem and look at extreme cases. Specifically, they look at how far Tommy could go if he biked on only the road or on only the grass. By doing so, the students were able to simplify the problem and only consider cases where Tommy's speed was constant throughout his entire trip.

2. If Tommy switches directions while on the grass, how does that impact how far he can go?

   If Tommy switches direction while on the grass, he will not be able to travel as far from the point he veered off the road as he would have if he continued in a straight path. For any point P that Tommy leaves the road, there is a circle of radius r that represents how far Tommy can go from point P. However, to achieve this maximum radius, he must continue in a straight line without switching direction. If he switches direction, Tommy will travel less far from the point P.

3. In line 11, Chris considers what happens if Tommy rides his bike on the road for 6 miles before riding on the grass. How far can he go on the grass from the point he veers off the road? How do you know this?

   In the Student Dialogue, students take the distance traveled on the road (i.e., 6 miles) and convert that into a time (i.e., 36 minutes) by thinking proportionally about the fact that Tommy rides 10 miles on the road in 60 minutes. Since Tommy has to be back home in 60 minutes and he spent 36 minutes on the road, that means he can ride on the grass for 24 minutes. Using proportional reasoning again and the fact that he can ride 6 miles on the grass in 60 minutes, that means he is able to drive 2.4 miles on the grass.

Possible Responses to Related Mathematics Tasks

1. Given the problem in the Student Dialogue, answer the following:
   A. If Tommy were to ride on the road for 1 mile before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   B. If Tommy were to ride on the road for 2 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   C. If Tommy were to ride on the road for 3 miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?
   D. If Tommy were to ride on the road for d miles before riding on the grass, how far could he go from the point he left the road and still make it back home in an hour?

      A. If Tommy rode on the road for 1 mile, that means he had been traveling for $\frac{1}{10}$ of an hour. Since he has $\left(1-\frac{1}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{1}{10}\right)\cdot 6$ or 5.4 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.

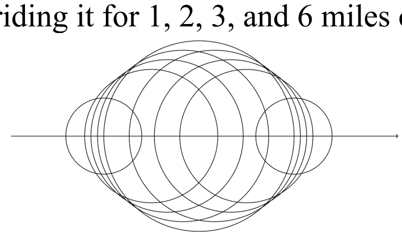
B. If Tommy rode on the road for 2 miles, that means he had been traveling for $\frac{2}{10}$ of an hour. Since he has $\left(1-\frac{2}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{2}{10}\right)\cdot 6$ or 4.8 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.

C. If Tommy rode on the road for 3 miles that means he had been traveling for $\frac{3}{10}$ of an hour. Since he has $\left(1-\frac{3}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{3}{10}\right)\cdot 6$ or 4.2 miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.

D. If Tommy rode on the road for d miles, that means he had been traveling for $\frac{d}{10}$ of an hour. Since he has $\left(1-\frac{d}{10}\right)$ hours left to ride on the grass, Tommy can travel a maximum distance of $\left(1-\frac{d}{10}\right)\cdot 6$ miles from the point he left the road. Tommy can travel anywhere on the grass inside a circle of that radius from the point he left the road.

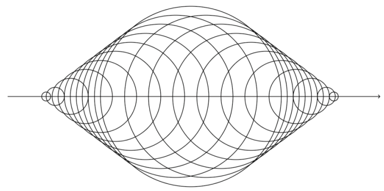
2. In Student Discussion Question 3 and Related Mathematics Task 1, you found the distance that Tommy could ride on the grass if he veers off the road after riding on the road for 1, 2, 3, and 6 miles. Draw a picture to scale representing the area that Tommy can ride on the grass after he veers off the road. (Or, better yet, create this picture using geometry software.) What do you notice about your picture? What conjecture can you make?

An image at scale of the circles representing the area Tommy can ride on the grass after he veers off the road after riding it for 1, 2, 3, and 6 miles can be seen below.



Modeling Problem—Biking Home

Looking at the image above, you might wonder about the shape of the boundary formed by the set of circles. This boundary is the outer perimeter of the region that Tommy can ride and still be back home within an hour, but what does it look like? If several circles are drawn accurately, then you may begin to notice that a straight line is formed along the edges. If the linear portion of this perimeter is difficult to see, you can draw more circles (to scale) if Tommy veers off the road after riding it for different distances. This picture with additional cases would be like the one below.



One conjecture that can be made is that the final solution to the problem can be drawn as follows: (1) draw a 6-mile circle around home, (2) mark down the two points along the road that are 10 miles away from home, and (3) form line segments tangent to the circle and connecting the two points on the road. This conjecture of the final solution would look like the image below.