Solving Problems by Creating Expressions—Dollar Bills

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Solving Problems by Creating Expressions—Dollar Bills* Illustration: This Illustration's student dialogue shows the conversation among three students who are trying to figure out what possible dollar amounts can be made using only one-dollar and five-dollar bills in a 3 to 1 ratio. After trying several numerical examples that fit the given conditions, students write an expression describing the possible dollar amounts that can be made.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 2: Reason abstractly and quantitatively.
MP 6: Attend to precision.
MP 8: Look for and express regularity in repeated reasoning.

Target Grade Level: Grades 6–7

Target Content Domain: Expressions and Equations

Highlighted Standard(s) for Mathematical Content

6.EE.A.2a Write, read, and evaluate expressions in which letters stand for numbers. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.*

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Math Topic Keywords: algebraic expressions

This material is based on work supported by the National Science Foundation under Grant No. DRL-1119163. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

If Raj has 3 times as many one-dollar bills as he has five-dollar bills, what are possible amounts of money he could have? Could he have $40? Could he have $42? What do you notice about the amounts he could have?

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Student Dialogue

Suggested Use
The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

Students in this dialogue have very basic familiarity with the practice of using one or more variables to represent unknown or changing values. They are now investigating a context where using variables will help them solve a problem. They understand the concept of multiples, whole numbers, and divisibility.

(1) Sam: I'm not sure where to start. Can we make some guesses and see what we get?

(2) Anita: Ok, let's say he has 4 one-dollar bills—that's $4. So, how many five-dollar bills does he have then?

(3) Dana: Well, the problem says he has, "3 times as many" so that's 12 five-dollar bills.

(4) Anita: But it says he has 3 times as many *ones* as he has fives. If he has 12 five-dollar bills, that's 3 times as many fives as he has ones. It's the other way around.

(5) Dana: Ok, he has 4 one-dollar bills and… wait. He can't have 4 one-dollar bills if it's 3 times as many! The number of bills is a whole number. We'd better pick the number of five-dollar bills first because the number of ones depends on that.

(6) Sam: Right, it's better to multiply the number of fives by 3 than to divide something like 4 by 3.

(7) Anita: So, what if he has 4 *five*-dollar bills? Then 3 times as many ones is $12. So plus $20 in five-dollar bills makes $32 total.

(8) Sam: We need more if we want to make $40: 5 five-dollar bills makes $25. He has 3 times as many ones, which is $15. And that makes $40 total.

(9) Anita: That answers one question! He *could* have $40! Let's try 6 five-dollar bills and use a table to keep track of everything.

[Anita draws the table below.]

Number of Five-Dollar Bills	Value from Five-Dollar Bills	Number of One-Dollar Bills	Value from One-Dollar Bills	Total Value of Money
4	$20	12	$12	$32
5	$25	15	$15	$40
6				

If he has 6 five-dollar bills, that's $30. Then 3 times as many ones is $18. And $30 + $18 = $48… Hey, we skipped $42! Is it possible to get $42?

(10) Sam: Between 5 and 6 fives, I guess, but we can't have a fraction of a bill.

(11) Anita: If we need $42, why don't we just add two more one-dollar bills to $40?

(12) Dana: But then it's not 3 times as many as the number of five-dollar bills.

(13) Anita: Oh right, so $42 isn't possible. What possible amounts of money could he have?

(14) Dana: Let's try a few more numbers of five-dollar bills and look for a pattern.

(15) Sam: I'll try 7. It's hard to work with, but it makes me think, and then I can see the pattern better.

(16) Anita: I'll try 10, because it's easy to multiply. I can do 20, too, by doubling Raj's money for 10 five-dollar bills.

(17) Dana: I guess I'll try 11 then.

[The students spend a few minutes to calculate the total for more numbers and fill in several rows of the table.]

Number of Five-Dollar Bills	Value from Five-Dollar Bills	Number of One-Dollar Bills	Value from One-Dollar Bills	Total Value of Money
4	$20	12	$12	$32
5	$25	15	$15	$40
6	$30	18	$18	$48
7	$35	21	$21	$56
10	$50	30	$30	$80
11	$55	33	$33	$88
20	$100	60	$60	$160

(18) Dana: Hey! Look at the possible totals. They are all even!

(19) Sam: It's more than that. They are all multiples of 8!

(20) Anita: Ok, so how can we think about this problem for *any* number of five-dollar bills and see if that's always true?

(21) Dana: Well, we'll need a variable…

(22) Sam: So, what was changing in each guess?

(23) Dana: Um, the number of fives, the number of ones, and the amount of money.

(24) Sam: I mean, what was the first thing that *we* changed each time and everything else changed based on that?

(25) Anita: Oh! We kept changing the number of five-dollar bills because the number of one-dollar bills *depends on* the five-dollar bills.

(26) Dana: Ok, let's use x as the variable for the number of five-dollar bills. That's the part that we don't know and want to figure out if it's a whole number.
[Dana starts another row in the table and writes x in the first column.]

(27) Sam: So, the amount of money from x five-dollar bills is $5x$ dollars.
[Sam writes $5x$ in the second column.]

(28) Anita: Aha! And there are 3 times as many one-dollar bills as five-dollar bills, so there are $3x$ one-dollar bills, and that's $3x$ dollars.
[Anita writes $3x$ in the third and fourth columns.]

(29) Sam: And added to the $5x$ makes $8x$ total dollars when we start with x five-dollar bills!
[Sam writes $8x$ in the last column.]

Number of Five-Dollar Bills	Value from Five-Dollar Bills	Number of One-Dollar Bills	Value from One-Dollar Bills	Total Value of Money
…	…	…	…	…
x	$5x$	$3x$	$3x$	$8x$

(30) Dana: Great! So… what does that mean?

(31) Anita: If we know the number of five-dollar bills, then we know the total amount?

(32) Sam: Yeah, but what does that say about what's possible? Why wasn't $42 possible?

(33) Dana: Oh! Since x is the number of five-dollar bills, it has to be a whole number because we can't rip money up. So, no matter what whole number we pick for x, we don't get $42.

(34) Anita: That's because the solution to $8x = 42$ isn't a whole number.

(35) Sam: Oh, and for possible amounts that he could have, like 40, we get an equation like $8x = 40$, which *does* have a whole number solution, 5!

(36) Dana: I guess that makes sense. We can see that the total is a multiple of 8 since it's equal to $8x$ and x is a whole number.

(37) Sam: So, multiples of 8 are the only possible amounts of money Raj could have under this condition. Oh, and they have to be positive, too.

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Teacher Reflection Questions

Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?

2. What strategy did students use to develop an algebraic expression?

3. How could you support students in using a similar strategy to solve this problem?

4. What other ways might students approach this mathematics task?

5. What does each of these expressions represent in the dialogue: x, $5x$, $3x$, $8x$?

6. What difficulties might students have understanding the meaning of the different coefficients in the algebraic expressions seen in the dialogue? How might you support students in understanding what the coefficients mean?

7. The language “3 times as many” can be confusing, and students may find it more natural to multiply the number of *one*-dollar bills by three (which is incorrect). How can you support students in working through this common misconception?

8. What if a student insists that x should be the number of one-dollar bills? What algebraic expression does this lead to? How can we see that it still needs to be a multiple of 8?

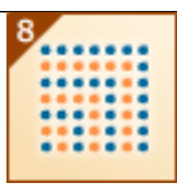
9. How could changing the multiplier, 3, in “3 times as many” affect the problem? What if it were 4 times as many or $\frac{1}{4}$ times as many?

10. How could changing the denominations of the bills affect the algebraic representation of the problem?

Mathematical Overview

Suggested Use
The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	**Evidence**
 Reason abstractly and quantitatively.	Students in this dialogue are using calculations and algebraic expressions in a thoughtful way to solve the problem. The students are "paus[ing] as needed during the manipulation process in order to probe into the referents for the symbols involved." Dana does this in line 5 when arguing why Raj can't have 4 one-dollar bills since that would result in a fractional number of five-dollar bills. Dana understands the constraints on an otherwise sensible calculation ($4 \div 3 = 1\frac{1}{3}$). Sam does this in line 10 as well when explaining how they could theoretically make \$42 based on the pattern seen in the students' table, but that the fractional number of five-dollar bills needed would be unrealistic.
 Attend to precision.	The students refine their communication as Dana suggests the use of a variable. Anita continues to clarify why it makes sense to change "the number of five-dollar bills because the number of one-dollar bills *depends on* the five-dollar bills" (line 25). The students are clear about the meaning of x (the number of five-dollar-bills) and why they selected that changing value for the variable (because everything else depends on that). Then, as students work through several expressions toward the final expression for the total amount of money, they describe the meanings of each expression and how each coefficient relates to the problem.
 Look for and express regularity in repeated reasoning.	In this dialogue, the students explore a few possibilities with numbers and identify patterns in the computations that lead to a solution. This is the process of generalizing from repeated reasoning. After several explorations with numbers, the students are able to use the calculation process that they have discovered to produce an algebraic expression by completing the same computational process with a variable input.

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Commentary on the Mathematics

Some mathematical problems do not have a straightforward solution process; thus, diving in and exploring possibilities is the right process. The students in this dialogue dive right into the mathematics and reason through the challenges they face as they see how the number of one-dollar bills depends on the number of five-dollar bills. Once they work through the calculation process with one number, they repeat it—finding a regularity in their reasoning and building confidence in that reasoning—until they are ready to complete the calculations with a variable representing the number of five-dollar bills.

Evidence of the Content Standards

This process supports students as they learn to write, read, and evaluate algebraic expressions (6.EE.2) and to use algebraic expressions and statements to solve problems (7.EE.4).

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Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. Why does the number of five-dollar bills in the dialogue have to be positive?
2. What does Anita mean that she "can do 20 [five-dollar bills], too, by doubling Raj's money for 10 five-dollar bills?" Does that work? Why? How could you check?
3. If Raj has a total of $96, how many of each bill does he have?
4. If Raj has a total of $32, how many of each bill does he have?
5. How do the students use the table to help them create the algebraic expressions for the problem?
6. What does the x represent in the dialogue?
7. What does the expression $5x$ represent?
8. What does the expression $3x$ represent?
9. What does the expression $8x$ represent?
10. Why do the students use only one variable? What if they were to use x for the number of five-dollar bills and y for the number of one-dollar bills?
11. How do these students decide which number to replace with a variable?

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Related Mathematics Tasks

1. Carmen has 2 five-dollar bills and 3 one-dollar bills, and Kareem has 3 five-dollar bills and 2 one-dollar bills. Who has more money?

2. If Raj has a total of $120, how many of each bill does he have?

3. What if Raj has **four** times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

4. What if Raj has **seven** times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

5. What if Raj had a times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

6. What if Raj had **four** times as many one-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

7. What if Raj has **four** times as many **ten**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

8. What if Raj has **seven** times as many **hundred**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

9. What if Raj has a times as many **hundred**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

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Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?

    Refer to the Mathematical Overview for notes related to this question.

2. What strategy did students use to develop an algebraic expression?

    The students guessed several numbers of five-dollar bills, checked the total value of the money in each case, and repeated the process enough times that they were able to perform the same set of calculations on a variable, x, and generalize to an algebraic expression for the total value of the money in any case.

3. How could you support students in using a similar strategy to solve this problem?

    Try having students complete the table for several more starting numbers (of five-dollar bills). Use non-consecutive numbers to establish the patterns across the rows rather than down the columns (e.g., $x = 7, 10, 20, 50, 100, 1000, \text{etc.}$). As students are able complete the calculation with a variety of numbers, ask them to consider how to fill in a row if the number of five-dollar bills was unknown and given as a variable. Have students clearly explain their thinking and the meaning of their expressions as they complete the same calculations on a variable resulting in the final row shown here:

Number of Five-Dollar Bills	Value from Five-Dollar Bills	Number of One-Dollar Bills	Value from One-Dollar Bills	Total Value of Money
4	$20	12	$12	$32
5	$25	15	$15	$40
6	$30	18	$18	$48
7	$35	21	$21	$56
10	$50	30	$30	$80
11	$55	33	$33	$88
20	$100	60	$60	$160
x	$5x$	$3x$	$3x$	$8x$

4. What other ways might students approach this mathematics task?

   Students may find it helpful to use visual methods (e.g., sketching pictures) or hands-on tools (e.g., pretend denominations) to discover the answer to this problem. Encourage your students to discuss their understanding of the problem, their questions about the dialogue, and other strategies for finding the solution.

5. What does each of these expressions represent in the dialogue: x, $5x$, $3x$, $8x$?

   Refer to the Student Discussion Questions for descriptions of each expression.

6. What difficulties might students have understanding the meaning of the different coefficients in the algebraic expressions seen in the dialogue? How might you support students in understanding what the coefficients mean?

   Students may find the different meanings of the coefficients in the expressions confusing. In $3x$, the coefficient 3 represents the ratio of one-dollar bills to five-dollar bills (the count of one-dollar bills), whereas the coefficient 5 represents the value of a five-dollar bill. When these expressions are combined into $8x$, the coefficient loses its concrete meaning and just becomes an indicator that the total amount of money will be 8 times the number of five-dollar bills. Students may find it surprising or hard to accept that the 8 doesn't have a meaning like the 3 and the 5.

   You may wish to address this potential confusion straight on by asking students to clarify and discuss the meaning of the coefficients in $3x$ and $5x$, and possibly also to consider what the 8 in $8x$ means. Or you might find it helpful to have students explain each numerical value in a few rows of the table describing the "why" of each answer and then have them explain the final row of the table with the variables. It might sound something like this: "From 4 five-dollar bills, you get $\$5 \cdot 4 = 20$ dollars, and there will be 12 one-dollar bills from $3 \cdot 4$, so you get \$12, and there is \$32 total, which is 8 times 4. From x five-dollar bills, you get $\$5 \cdot x = 5x$ dollars, and there will be $3x$ one-dollar bills (from $3 \cdot x$), so you get $3x$ dollars, and there is $8x$ total, which is 8 times x or just $8x$." Challenge students to explain what the 8 in $8x$ means. All it means is that the total money is 8 times the number of five-dollar bills. Sometimes, hearing the reasoning several times from different perspectives and reasoning out loud themselves can help students understand.

7. The language "3 times as many" can be confusing, and students may find it more natural to multiply the number of *one*-dollar bills by three (which is incorrect). How can you support students in working through this common misconception?

   Exploring several numerical examples can make this idea more clear. For example, if there are 5 five-dollar bills, how many one-dollar bills will there be?

8. What if a student insists that x should be the number of one-dollar bills? What algebraic expression does this lead to? How can we see that it still needs to be a multiple of 8?

   One approach to working with this suggestion would be to work out that the number of five-dollar bills would be $\frac{x}{3}$, and hence the value of all the five-dollar bills would be $\frac{5x}{3}$. Then, adding the value of the x one-dollar bills gives $\frac{5x}{3}+x=\frac{8x}{3}$. We know $\frac{x}{3}$ is an integer because it's the number of five-dollar bills, so $\frac{8x}{3}$ has to be a multiple of 8. This could be interesting to demonstrate and validates Dana's suggestion that they'd "better pick the number of five-dollar bills first."

9. How could changing the multiplier, 3, in "3 times as many" affect the problem? What if it were 4 times as many or $\frac{1}{4}$ times as many?

   This changes the coefficient in the expression that represents the total value from one-dollar bills. If Raj had 4 times as many ones as fives, the total amount of money from all bills would be $4x+5x=9x$. The only amounts of money he could make would be multiples of 9. If he had $\frac{1}{4}$ as many ones as he had fives, the total amount would be $\frac{1}{4}x+5x=5\frac{1}{4}x$. Then the amount of five-dollar bills would have to be a multiple of 4, and the only amount of money would be multiples of $5\frac{1}{4}\cdot 4$ or 21. Or if Raj had any number, a, times as many, the total amount of money from all bills would be $ax+5x$, so the only amounts of money he could make would be multiples of $(a+5)$. There are problems connected to this idea in Related Mathematical Tasks.

10. How could changing the denominations of the bills affect the algebraic representation of the problem?

    Changing the denomination of the five-dollar bills would change the coefficient of the expression $5x$. For example, for fifty-dollar bills, it would be $50x$, or for c-dollar bills, it would be cx.

    Changing the denomination of the one-dollar bills would add another factor to the $3x$ term. For example, for ten-dollar bills, it would be $30x$, or for b-dollar bills, it would be $3bx$.

    If variables are used, factoring the expression that says what amounts of money can be made gives more insight into those amounts than leaving the expression unfactored. For example, if Raj has 3 times as many ten-dollar bills as he has fifty-dollar bills, the expression for the total amount of money would be $80x$, but if Raj has 3 times as many b-dollar bills as he has c-dollar bills, the expression for the total amount of money would be $3bx + cx$, and we get more of a sense of what can be made by seeing that expression as a multiple of x: in particular, it says that any multiple of $3b + c$ can be made.

Possible Responses to Student Discussion Questions

1. Why does the number of five-dollar bills in the dialogue have to be positive?

   The number of five-dollar bills should be positive because while you can have negative money if you have debt, you cannot have negative dollar bills.

2. What does Anita mean that she "can do 20 [five-dollar bills], too, by doubling Raj's money for 10 five-dollar bills?" Does that work? Why? How could you check?

   Anita has noticed that doubling the input will double the output. This occurs because there is only multiplication in the corresponding expression $8x$. With expressions like $8x + 3$, $8(x + 5)$, or even just $x + 7$, this would not be the case. A simple way to check that doubling the input will double the output is to calculate the total amount of money for double the number of five-dollar bills ($2x$). Then, there will be $10x$ dollars from five-dollar bills and 3 times as many one-dollar bills as there are five-dollar bills ($6x$), so $6x$ dollars from one-dollar bills, and the total amount will be $16x$, which is double the output.

3. If Raj has a total of \$96, how many of each bill does he have?

   Following the logic of the students in the dialogue, we see that the total value of the money (\$96) is 8 times the number of five-dollar bills, so $96 \div 8 = 12$. So there are 12 five-dollar bills and 3 times as many (36) one-dollar bills. Alternatively, students may see that $8x = 96$ means that $x = 12$.

4. If Raj has a total of \$32, how many of each bill does he have?

   Raj would have 4 five-dollar bills and 12 one-dollar bills.

5. How do the students use the table to help them create the algebraic expressions for the problem?

   They keep track of each calculation for each number they try. Then, they use the table to track their thinking as they work through the same calculation for an unknown number of five-dollar bills.

6. What does the x represent in the dialogue?

   The students chose x to represent the number of five-dollar bills.

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7. What does the expression $5x$ represent?

   The $5x$ represents the amount of money from x five-dollar bills.

8. What does the expression $3x$ represent?

   The $3x$ represents the amount of money from the one-dollar bills since there are 3 times as many one-dollar bills as there are five-dollar bills.

9. What does the expression $8x$ represent?

   The expression $8x$ represents the total value of the money Raj has. It comes from the amount of money from five-dollar bills, $5x$, and the amount of money from one-dollar bills, $3x$, added together.

10. Why do the students use only one variable? What if they were to use x for the number of five-dollar bills and y for the number of one-dollar bills?

    The students decide it is better to choose the number of $5 bills and calculate the number of one-dollar bills, $3x$, based on the number of fives because the value and number of one-dollar bills depend on the number of five-dollar bills, and the value of the five-dollar bills, $5x$, does too.

    If they had used y for the number of one-dollar bills, the total value of the money would be $5x+y$, which cannot be combined, and there is still more information to use. Since Raj has 3 times as many one-dollar bills as he has five-dollar bills, y would equal $3x$ (trying examples can make that idea more clear). Then the total value of money is really $5x+3x$, which is $8x$ in the end anyway, just with extra work to get there.

11. How do these students decide which number to replace with a variable?

    The students decide that it is better to choose a number of $5 bills and multiply by 3 to get the number of $1 bills than to choose the $1 bills first and divide by 3, because dividing could give them a fraction of a bill, which doesn't make sense.

Possible Responses to Related Mathematics Tasks

1. Carmen has 2 five-dollar bills and 3 one-dollar bills, and Kareem has 3 five-dollar bills and 2 one-dollar bills. Who has more money?

   Carmen has $2\cdot\$5=\$10$ from five-dollar bills and $3\cdot\$1=\$3$ from one-dollar bills, which makes $13. Kareem has $3\cdot\$5=\$15$ from five-dollar bills and $2\cdot\$1=\$2$ from one-dollar bills which makes $17. Kareem has more money.

2. If Raj has a total of \$120, how many of each bill does he have?

   The total value (\$120) is 8 times the number of five-dollar bills, so $120 \div 8 = 15$. So he has 15 five-dollar bills and 3 times as many (45) one-dollar bills.

3. What if Raj has **four** times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $4x + 5x = 9x$, so Raj could have any whole number multiple of 9 dollars.

4. What if Raj has **seven** times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $7x + 5x = 12x$, so Raj could have any whole number multiple of 12 dollars.

5. What if Raj had a times as many one-dollar bills as he has five-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $ax + 5x = (a+5)x$, so Raj could have any whole number multiple of $a + 5$ dollars.

6. What if Raj had **four** times as many one-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $4x + 50x = 54x$, so Raj could have any whole number multiple of 54 dollars.

7. What if Raj has **four** times as many **ten**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $4 \cdot 10x + 50x = 90x$, so Raj could have any whole number multiple of 90 dollars.

8. What if Raj has **seven** times as many **hundred**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $7 \cdot 100x + 50x = 750x$, so Raj could have any whole number multiple of 750 dollars.

9. What if Raj has a times as many **hundred**-dollar bills as he has **fifty**-dollar bills? What possible amounts of money could Raj have under this condition?

   The expression for the total amount of money would be $100ax + 50x = (100a + 50)x$, so Raj could have any whole number multiple of $100a + 50$ dollars.