Adding Fractions with Unlike Denominators

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Adding Fractions with Unlike Denominators* Illustration: This Illustration's student dialogue shows the conversation among three students making sense of what it means to add fractions with unlike denominators. While the students do not finish developing an algorithm for adding such fractions, they do come to understand that a common "unit" is needed.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 1: Make sense of problems and persevere in solving them.
MP 2: Reason abstractly and quantitatively.
MP 3: Construct viable arguments and critique the reasoning of others.
MP 6: Attend to precision.

Target Grade Level: Grades 4–5

Target Content Domain: Number and Operations—Fractions

Highlighted Standard(s) for Mathematical Content

4.NF.B.3a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Math Topic Keywords: fractions, common denominator, unlike denominator, adding fractions, equivalent fractions

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Adding Fractions with Unlike Denominators

Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

What does $\frac{2}{5}+\frac{1}{2}$ equal?

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Student Dialogue

Suggested Use
The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

The students in this dialogue have experience comparing fractions and adding fractions that have the same denominator. In this dialogue, they are beginning to reason about the meaning of adding fractions with unlike denominators in the context of an example problem.

(1) Sam: How do you do $\frac{2}{5} + \frac{1}{2}$?

(2) Dana: It's just $\frac{3}{7}$, isn't it?

(3) Anita: But $\frac{3}{7}$ is *less* than $\frac{1}{2}$, so it can't be that!

(4) Sam: So… how do you do it?

(5) Dana: But we're just adding: 2 + 1 is 3, and 5 + 2 is 7, so it *should be* $\frac{3}{7}$.

(6) Anita: We already know that 2 fifths plus 1 fifth is 3 fifths *[writes down* $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$*]*. It's not 3 tenths. You can't just add everything you see.

(7) Sam: So… how *do* you do it?

(8) Dana: *[to Anita]* Oh, right, I get it. It's like when we were saying "2 cats plus 1 cat, 2 grapes plus 1 grape, 2 fifths plus 1 fifth."

(9) Sam: Yeah, I get it, *too*, but how do we do 2 *fifths* plus 1 *half*?! It's *not* just 3 of something, but what *is* it? We're adding two different things. Like 2 cats and 1 grape; 2 feet and 1 inch. Or, maybe like 2 thousand and 1 hundred. We *can* add them, but they're not *three* of something.

(10) Dana: Oh, we need the same thing. 2 feet and 1 inch is 25 inches. Or, 2 thousand plus 1 hundred is like saying 20 hundreds plus 1 hundred. *[writes down 2100]* That's 21 hundred.

(11) Anita: Or 2.1 thousand: 2 thousand plus 0.1 of a thousand. But not 3 of anything.

(12) Sam: *[sighs]* Yeah, but I still don't know how to do $\frac{2}{5} + \frac{1}{2}$…

(13) Dana: So, we need to make the fifths and the halves the same somehow so we can add them together more easily.

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Teacher Reflection Questions

Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

2. How is spoken language helpful when working with fractions with unlike denominators?

3. What's wrong with the claim "you can't add $3a^2$ and $2a$"?

4. Students might still try to add 2 cats and 1 grape saying it's 3 objects. Adding 2 thousand and 1 hundred together makes it harder to see some sensible common unit. What other examples might be good to use to show the logic of needing a common unit?

5. At the end of the dialogue, Sam still doesn't know how to add $\frac{2}{5}+\frac{1}{2}$. What does the student need to understand in order to add those two fractions? How could you help Sam build that understanding?

6. What other common denominators would work? Sam could explore what the resulting fractions would be with different common denominators. What happens if 100 is used as the common denominator? What if 5 is the common denominator?

7. Are there *any* fractions whose (correct) sum can be found by adding the numerators and adding the denominators? How do you know?

8. When adding, it is important to have a common unit. What roles do units play in multiplication?

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Mathematical Overview

Suggested Use The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	Evidence
Make sense of problems and persevere in solving them.	In lines 6–11 of this dialogue, the students "consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution" (MP 1). They discuss examples of adding fifths, cats, grapes, cats and grapes, feet and inches, and thousands and hundreds as they try to make sense of what it means to add two fractions with unlike denominators.
Reason abstractly and quantitatively.	The cats and grapes argument (lines 8–9) uses a totally non-numeric context to make sense of the numeric situation and to understand how the notion of a common unit might apply to fractions. Students are "considering the units involved; attending to the meaning of quantities, not just how to compute them" (MP 2). When we add two numbers, we tacitly assume that the numbers quantify the same thing. As Sam points out in (line 9), we *can* add 2 thousand and 1 hundred, but it is not *3* of any thing we've already named. The same is true of $2x + 3y$. We can add them—in fact, we just did—but they are not 5 of anything we've already named. (What *are* they five of? What are $\frac{2}{5} + \frac{1}{2}$ *three* of? What is $\frac{a}{b} + \frac{c}{d}$ "a + c" of?) In this dialogue, the students never figure out how to convert the fractions into forms (e.g., 4 tenths and 5 tenths) that give them integer counts (4 and 5) that they can add, but they do establish what is needed (Dana says, "the same thing" (lines 10, 13), and they do establish that without that, they cannot simply add numerators (let alone denominators).
Construct viable arguments and critique the reasoning of others.	The students are not exactly constructing and critiquing arguments in the sense implied in MP 3, in that they are not really developing much of a chain of reasoning—during the course of this dialogue they do not reach a conclusion about what they should do, only that they should not add the fractions as if the denominators were the same units. However, the students are beginning to engage in MP 3 in the conjectures they are making and their evaluation of each others' conjectures. For example, Dana thinks that they can add fractions by adding the numerators and

	denominators, but Anita (lines 3, 6) evaluates the reasonableness of this claim by comparing $\frac{3}{7}$ to $\frac{1}{2}$ in one case, and by providing a counter-example in another case (i.e., "$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$. It's not 3 tenths."). All of the students are flirting with this practice of constructing arguments by their increasingly clear verbalization and their *attention* to each others' statements.
 Attend to precision.	The students in this dialogue work to communicate precisely with each other (MP 6) by jointly establishing what units they are talking about when answering the stated problem and how those units impact what they can do. Starting in line 8, the three students begin to highlight for each other how the units for the two fractions being added are different from each other, giving specific examples of why defining the units will matter when trying to add the fractions.

Commentary on the Mathematics

The dialogue also illustrates the idea of students *working through* a problem for which none of them have a complete solution, and *checking* their answers rather than just jumping to a solution. Importantly, the students don't fall back on a rule to provide an answer. The discussion of twenty-one hundred and 2.1 thousand both hint at the ideas underlying a method for combining the two addends—the need for a common unit—as does their appeal to the cats-and-grapes argument, but these all draw on students' understanding and "sense making": they are attempts to *derive* a logical method, rather than attempts to *recall* a learned (but maybe not understood) rule about "common denominators." Students push each other to think through why the temptation to add both the numerators and the denominators—a common misconception—will not work. Though this little clip of dialogue ends before the students have solved their problem (see Sam's disappointment in line 12), the dialogue represents an important step in learning to persevere.

At some point, the *content* goal—figuring out how to add the two fractions—still needs to be satisfied, and waiting *too* long without resolution risks frustration and loss of interest, but if teaching brings closure too soon or too often, and if too many problems are resolved quickly, students don't get a chance to stretch their ability to struggle. From line 9 on, we get many hints that they're close to a resolution and might, in the next few minutes, hit on it by themselves. More likely, they will need help, but, having struggled with the problem and come this far, they're now really ready for that help. The teacher who is lucky enough to have overheard their reasoning is now in a perfect position. They've made the necessary steps, and may just not realize (or have the confidence to believe in) the value of what they've done. As Dana says, the fractions need to be made "the same somehow," and the statements by all three in lines 9 through 11 give exactly the right ideas. $\frac{2}{5} + \frac{1}{2}$ is not three of something, but if *both* of those fractions were made "the same somehow" by changing both into tenths or twentieths or, for that matter, fifths…. The teacher's role may be as simple as saying: "Exactly! So how can you do that?!"

Adding Fractions with Unlike Denominators

Evidence of the Content Standards

Although the students in the dialogue do not reach the correct algorithm, they explore what it means to add two fractions with unlike denominators (5.NF.B.3a). Initially the students apply a common incorrect algorithm but quickly realize they are mistaken by comparing their answer to the benchmark fraction ½ (line 3) (5.NF.A.2). Later the students talk about how adding fractions, they need to add or join two quantities of the same unit (lines 6–8) (4.NF.B.3a) and continue to explore the implications of this in the remainder of the dialogue.

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Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. In the student dialogue, how does Dana get $\frac{3}{7}$ from $\frac{2}{5}+\frac{1}{2}$? What is the common mistake the student is making?

2. In line 3, how does Anita realize that $\frac{3}{7}$ is incorrect? How does Anita know this?

3. In line 8, Dana is giving several examples of addition. What do you notice about all the examples? What does this tell you about two numbers if you want to add them together to get one number?

4. Using a number line, how can you think of fifths and halves as units?

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Related Mathematics Tasks

1. Consider the following example:

   A. What is 2 quarts plus 1 cup? James claims he can add these together to get 3 quarts of milk. Is this true? Why or why not?

   B. If 4 cups are in 1 quart, how many cups are in 2 quarts? What is 2 quarts plus 1 cup?

   C. Why could you add 2 quarts and 1 cup in part B but not in part A?

2. Consider the following example (Note: allons, bobbers, and coffs are made-up words):

   A. What is 3 allons plus 5 bobbers? Can you add the two numbers together, why or why not?

   B. If there are 5 coffs in an allon and 10 coffs in a bobber, what is 3 allons plus 5 bobberss?

   C. Why could you add 3 allons plus 5 bobbers in part B but not in part A?

3. Consider the following example:

   A. Is 2 fifths plus 1 half equal to 3 sevenths? Why or why not?

   B. If there are 2 tenths in 1 fifth and 5 tenths in 1 half, what is 2 fifths and 1 half?

   C. What did you do in part B that made it easier to add 2 fifths and 1 half?

   D. Write a word problem that would require you to add 2 fifths and 1 half and explain how you would solve the problem.

   E. Rewrite all the conversions and work you did in part B using fraction notation.

Adding Fractions with Unlike Denominators

Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

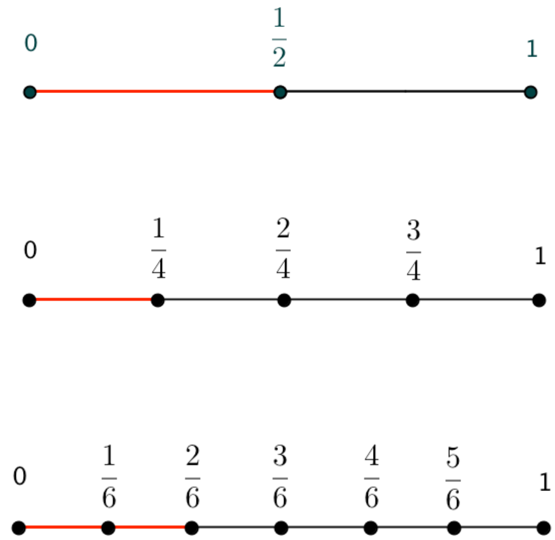
1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

    Refer to the Mathematical Overview for notes related to this question.

2. How is spoken language helpful when working with fractions with unlike denominators?

    Spoken language lets us hear the denominator as if it's a "thing." First graders who have no idea what "fifths" are will be sure, when the question is spoken, that 3 of them plus 3 of them will be 6 of them, because the "things" are the same. Similarly, when the question is spoken, they'll think it's a joke or look puzzled if asked what 3 potatoes plus 3 minutes is. It makes no sense. Written fraction notation, though, tempts students—if they don't already have the ideas very clear in mind—to "add everything in sight." So, the same first graders who know that "3 fifths plus 3 fifths" makes "6 fifths" might well, three years later, do the standard wrong thing with $\frac{3}{5}+\frac{3}{5}$. Similarly, hearing "1 half plus 1 fourth" makes clear that we can't just say "2" somethings, but seeing $\frac{1}{2}+\frac{1}{4}$ can tempt students, unless they are secure in their understanding, to write $\frac{2}{6}$ without thinking about what it means. Students are far less likely to answer the spoken question "What is 1 half plus 1 fourth?" with "2 sixths."

Plotting fractions on the number line, it becomes clear why $\frac{1}{2}+\frac{1}{4}\neq\frac{2}{6}$ since the line segments formed (shown in red) don't add up.



3. What's wrong with the claim "you can't add $3a^2$ and $2a$"?

   The claim "You can't add $3a^2$ and $2a$" is incorrect: $3a^2$ and $2a$ can be added, and the expression $3a^2 + 2a$ is the result. Another way to write the result is $a(3a+2)$. In fact, the latter uses exactly the same reasoning we use in order to combine $3b + 2b$ as $5b$. The mathematical step—though rarely the way we think about it intuitively—is factoring $3b + 2b$ as $b(3 + 2)$. What the claim "You can't add $3a^2$ and $2a$" really means is "you can't just add the coefficients" by themselves, because the objects that the coefficients "count" are like cats and grapes, or halves and fifths. A particularly good example is $3a + 3b$: we can't "add the numbers" to get 6 something or other, but we can add the variables to get $3(a + b)$. Similarly, in this dialogue, the students realize that for the problem 2 fifths plus 1 half, they cannot just add the coefficients 2 and 1 because these coefficients are "counting" different objects, but this does not mean that the two quantities cannot be added.

4. Students might still try to add 2 cats and 1 grape saying it's 3 objects. Adding 2 thousand and 1 hundred together makes it harder to see some sensible common unit. What other examples might be good to use to show the logic of needing a common unit?

   Examples from measurement are often convincing and pull for using sense rather than arbitrary (typically wrong) rules. 3 feet plus 2 yards pulls for a common unit; 2 hours plus 30 minutes isn't 32 of anything, but it is 150 minutes or 2½ hours; 1 cup plus 4 ounces isn't 5 of anything. Also, the hundreds and thousands work well because the arithmetic is so familiar. The reason "2 thousand" plus "1 hundred" isn't 3 of something is not just because when we write it "we line it up," but because even when it isn't written, we would never expect it to be 3 of anything!

5. At the end of the dialogue, Sam still doesn't know how to add $\frac{2}{5}+\frac{1}{2}$. What does the student need to understand in order to add those two fractions? How could you help Sam build that understanding?

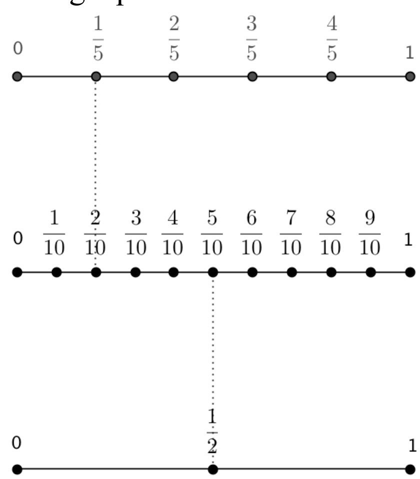
   Sam's comments in line 9 of the dialogue seem to indicate a clear understanding about adding like-denominator fractions, but it's worth starting there and checking. If, for example, Sam understands that $\frac{3}{5}+\frac{3}{5}$ is "6 fifths"—then the question about how to add $\frac{2}{5}+\frac{1}{2}$ makes it clear that Sam already understands why this situation is different. So, the remaining step is to replace one (or both) of these fractions with other fractions of equal value to allow the student to add. That is, Sam needs equivalent fractions with a common denominator. The "standard" method is to find a number that has both 5 and 2 as a factor (like 10 or 20 or 100) and then rewrite both fractions with that denominator. But the logic doesn't depend on the conventional method. With a suitable diagram, Sam could, for example, notice that $\frac{1}{2}$ is "$2\frac{1}{2}$ fifths." By this logic, then, $\frac{2}{5}+\frac{1}{2}$ would be "$4\frac{1}{2}$ fifths," which Sam can even write, same as any other number of fifths: $\frac{4\frac{1}{2}}{5}$. And, converting that to an equivalent fraction (say, by doubling top and bottom), can produce the "conventional" answer.

6. What other common denominators would work? Sam could explore what the resulting fractions would be with different common denominators. What happens if 100 is used as the common denominator? What if 5 is the common denominator?

   Sam's comments in line 9 of the dialogue seem to indicate a clear understanding about adding like-denominator fractions, but it's worth starting there and checking. If, for example, Sam understands that $\frac{3}{5}+\frac{3}{5}$ is "6 fifths"—then the question about how to add $\frac{2}{5}+\frac{1}{2}$ makes it clear that Sam already understands why this situation is different. So, the remaining step is to replace one (or both) of these fractions with other fractions of equal value to allow the student to add. That is, Sam needs equivalent fractions with a common denominator. The "standard" method is to find a number that has both 5 and 2 as a factor (like 10 or 20 or 100) and then rewrite both fractions with that denominator. But the logic doesn't depend on the conventional method. With a suitable diagram, Sam could, for example, notice that $\frac{1}{2}$ is "$2\frac{1}{2}$ fifths." By this logic, then, $\frac{2}{5}+\frac{1}{2}$ would be "$4\frac{1}{2}$ fifths,"

which Sam can even write, same as any other number of fifths: $\frac{4\frac{1}{2}}{5}$. And, converting that to an equivalent fraction (say, by doubling top and bottom), can produce the "conventional" answer.

One way Sam could be supported to find a common denominator is by providing several number lines, one on top of another, each plotting fractions of a different denominator (see example below, using 10 as a common denominator). Using number lines, students can visually see equivalent fractions and how to convert them so that the fractions they add have a common "unit" (i.e., denominator). Once Sam conceptually understands what equivalent fractions are and why they are necessary when adding/subtracting fractions, the numerical algorithm for writing equivalent fractions can be shown.



7. Are there *any* fractions whose (correct) sum can be found by adding the numerators and adding the denominators? How do you know?

We know that $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$. If it also equals $\frac{a+c}{b+d}$, then $\frac{ad+bc}{bd}=\frac{a+c}{b+d}$. So the next step is to see whether any integer solutions fit that. Multiplying both sides of the equation by bd reduces the equation to:

$$ad+bc=\frac{abd+cbd}{b+d}$$

And then multiplying by the remaining denominator $(b+d)$ gives the equation:

$$(ad+bc)(b+d)=abd+cbd$$

$$abd+ad^2+cb^2+cbd=abd+cbd$$

Next, subtract abd and cbd from both sides to get:

$$ad^2 + cb^2 = 0$$
$$ad^2 = -cb^2$$
$$\frac{a}{-c} = \frac{b^2}{d^2}$$

At this point, choose values for *b* and *d*, say $b = 2$ and $d = 3$. This would mean:

$$\frac{a}{-c} = \frac{2^2}{3^2}$$
$$\frac{a}{-c} = \frac{4}{9}$$

so $a = 4$ and $c = -9$.

To check that the two fractions, $\frac{4}{2}$ and $\frac{-9}{3}$, can have their numerators and denominators added and still give the correct sum, see if the fractions make the following true $\frac{ad+bc}{bd} = \frac{a+c}{b+d}$, in other words, that the correct and incorrect algorithms both give the same result in this case. Below is the check:

$$\frac{(4)(3)+(2)(-9)}{(2)(3)} = \frac{4+-9}{2+3}$$
$$\frac{12+-18}{6} = \frac{-5}{5}$$
$$\frac{-6}{6} = \frac{-5}{5}$$
$$-1 = -1$$

Are there other possibilities?

8. When adding, it is important to have a common unit. What roles do units play in multiplication?

The two numbers in multiplication sometimes play different roles, with one of them being a scale factor and the other counting or measuring something. For example, if we triple our money, the 3 in 3 × 10 dollars is a scalar, while the 10 counts dollars. Halving a length works the same way: $\frac{1}{2}$ × 8 gives 4. Though that example is about length, scaling is often well visualized as a change in length, even when the units aren't actually length. We can multiply cows by a number—we can scale up (or down) our herd of cattle—but we cannot sensibly multiply cows by cows.

However, units can be multiplied by other units sometimes: multiplication is not just about scaling a quantity. Sometimes it changes the kind of quantity. For example, inches can be multiplied not only by a scale factor like $\frac{1}{2}$—a change from one amount of inches to another amount of inches—but by other inches. In that case, we're changing from two amounts of inches to one amount of square inches (area). Combined units like foot-pounds or miles per second are cases of multiplying or dividing mixed units.

Possible Responses to Student Discussion Questions

1. In the student dialogue, how does Dana get $\frac{3}{7}$ from $\frac{2}{5}+\frac{1}{2}$? What is the common mistake the student is making?

   Dana added the numerators and denominators of the two fractions. This is a mistake since the student can't simply add the numerators and denominators. Dana is not considering that fifths and halves are different sized units that are being added together (2 of the fifths and 1 of the halves).

2. In line 3, how does Anita realize that $\frac{3}{7}$ is incorrect? How does Anita know this?

   Anita realizes that $\frac{3}{7}$ is less than $\frac{1}{2}$. The student may know this by realizing that the numerator 3 is less than half of 7 (in fractions equivalent to $\frac{1}{2}$ the numerator is exactly half the denominator). Since $\frac{3}{7}$ is less than $\frac{1}{2}$, it can't be the sum of $\frac{2}{5}$ and $\frac{1}{2}$ because you would expect the sum to be bigger than $\frac{1}{2}$. If you are adding a positive number (like $\frac{2}{5}$) to $\frac{1}{2}$, the final sum would be bigger not smaller.

3. In line 8, Dana is giving several examples of addition. What do you notice about all the examples? What does this tell you about two numbers if you want to add them together to get one number?

   Both summands have the same unit of measure. Both numbers are either representing number of cats or grapes or fifths.

4. Using a number line, how can you think of fifths and halves as units?

   You can think of fifths and halves as the units into which you are breaking up whole numbers. For example, on the number line it would take 5 fifths to go from 0 to 1 and another 5 fifths to go from 1 to 2, etc. Or, it would take 2 halves to go from 0 to 1 and another 2 halves to go from 1 to 2, etc. Using fractional parts as units is similar to breaking a meter stick into centimeters.

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Possible Responses to Related Mathematics Tasks

1. Consider the following example:

   A. What is 2 quarts plus 1 cup? James claims he can add these together to get 3 quarts of milk. Is this true? Why or why not?

      You can't add quarts and cups directly because they are different sizes (different units of measure).

   B. If 4 cups are in 1 quart, how many cups are in 2 quarts? What is 2 quarts plus 1 cup?

      There are 8 cups in 2 quarts. 2 quarts (which is the same as 8 cups) plus 1 cup will equal 9 cups.

   C. Why could you add 2 quarts and 1 cup in part B but not in part A?

      We were able to add 2 quarts to 1 cup in part B because we first converted quarts to cups. Once all the quantities are in cups they can simply be added together.

2. Consider the following example (Note: allons, bobbers, and coffs are made-up words):

   A. What is 3 allons plus 5 bobbers? Can you add the two numbers together, why or why not?

      You can't since they have different units (allons vs. bobbers).

   B. If there are 5 coffs in an allon and 10 coffs in a bobber, what is 3 allons plus 5 bobberss?

      3 allons = 15 coffs.
      5 bobbers = 50 coffs.
      This means 3 allons plus 5 bobbers equals 65 coffs.

   C. Why could you add 3 allons plus 5 bobbers in part B but not in part A?

      You could add allons and bobbers in part B because we were able to first convert both quantities to a common unit, coffs.

3. Consider the following example:

   A. Is 2 fifths plus 1 half equal to 3 sevenths? Why or why not?

      We can't add 2 fifths and 1 half to get 3 of some unit (e.g., sevenths) because they have different "units." One quantity is measured in fifths while the other is measured in halves.

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B. If there are 2 tenths in 1 fifth and 5 tenths in 1 half, what is 2 fifths and 1 half?

    2 fifths = 4 tenths
    1 half = 5 tenths.
    This means 2 fifths plus 1 half equals 9 tenths.

C. What did you do in part B that made it easier to add 2 fifths and 1 half?

    We were able to add 2 fifths and 1 half in part B because we first converted the quantities to a common unit, tenths.

D. Write a word problem that would require you to add 2 fifths and 1 half and explain how you would solve the problem.

    One example: John and his sister shared a candy bar. John ate 2 fifths of the candy bar and his sister ate 1 half. How much of the candy bar did they eat all together? To solve this, you need to be able to add the fifths and the half together. Each fifth is the same as 0.2, and each half is the same as 0.5. So, together they ate 0.2 + 0.2 + 0.5 = 0.9.

E. Rewrite all the conversions and work you did in part B using fraction notation.

$$\frac{2}{5} = \frac{4}{10}$$
$$\frac{1}{2} = \frac{5}{10}$$

This means:
$$\frac{2}{5} + \frac{1}{2} =$$
$$\frac{4}{10} + \frac{5}{10} = \frac{9}{10}$$