Comparing Fractions

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Comparing Fractions* Illustration: This Illustration's student dialogue shows the conversation among three students who are exploring ways to compare the size of two fractions. For different pairs of fractions they use methods including comparing the size of unit fractions with the same denominators as the pair of fractions; comparing the fractions to the benchmark fraction of 1/2; determining how close to 1 each fraction is, again using the idea of the size of unit fractions with the same denominators; and using a common denominator.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 2: Reason abstractly and quantitatively.
MP 3: Construct viable arguments and critique the reasoning of others.
MP 5: Use appropriate tools strategically.
MP 7: Look for and make use of structure.

Target Grade Level: Grades 4–6
Target Content Domain: Number and Operations—Fractions, The Number System

Highlighted Standard(s) for Mathematical Content

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Math Topic Keywords: interpreting and comparing fractions, fractions as division, number lines

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Comparing Fractions

Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

Compare each pair of fractions and decide which is the larger number:

a) 3/4 and 3/5
b) 3/7 and 5/8
c) 4/5 and 6/7
d) 4/5 and 5/7

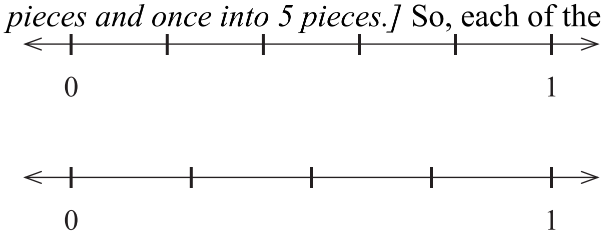
Student Dialogue

> **Suggested Use**
> The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

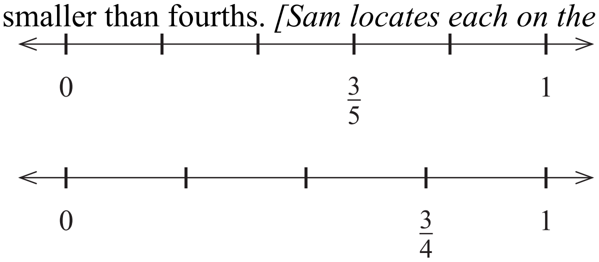
Students have experience interpreting fractions as division of the numerator by the denominator, and have been learning to evaluate fractions by placing them on the number line. Students have learned to interpret any fraction $\frac{n}{d}$ *as n times as much as the unit fraction* $\frac{1}{d}$*. That is, they look at the interval on the number line from 0 to 1, split that interval evenly as indicated by the denominator d, and then use the numerator to determine how many of those pieces to count up from zero.*

(1) Sam: Hmmm, three-fourths and three-fifths? Couldn't we use a number line or something…

(2) Dana: Well, we know how to compare *one*-fourth and *one*-fifth. One-fourth has to be bigger because it cuts 1 into only 4 pieces, and one-fifth cuts it into more pieces. *[Dana partitions the distance between 0 and 1 on two number lines: once into 4 pieces and once into 5 pieces.]* So, each of the one-fifth pieces will be smaller.



(3) Sam: Yeah, ok, so then three-fifths will be smaller than three-fourths because fifths are smaller than fourths. *[Sam locates each on the number line.]*



(4) Anita: Ok, that's done. Three-fourths is larger. How about Part b: three-sevenths and five-eighths?

(5) Sam: Can we do the same thing? A seventh is larger than an eighth…

(6) Dana: But we aren't counting the *same number* of sevenths and eighths. I mean, it makes sense that three-sevenths would be larger than three-eighths, but maybe *five*-eighths is *bigger* than three-sevenths.

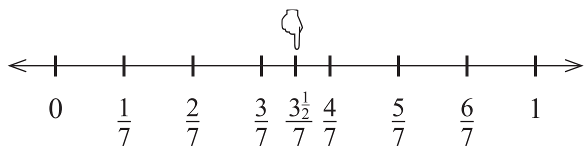
(7) Sam: We could put them on a number line…

(8) Anita: Yeah, but it's so hard to draw the number lines perfectly enough to be sure. Maybe we could just *think* about where they are on the number line…

(9) Dana: Well, five-eighths is a little more than a half because four-eighths *is* one-half.

(10) Sam: But with sevenths, it's hard to think about one-half because seven is odd. Oh wait, half of seven is between three and four. In fact, it's exactly three-and-a-half.

(11) Anita: Oh! So three-and-a-half-sevenths is one half. *[Anita writes* $\frac{3\frac{1}{2}}{7} = \frac{1}{2}$*.]* I've never seen a fraction like that before, but it makes sense.



(12) Dana: Uh, what are we doing again? Don't we need to compare three-sevenths and five-eighths?

(13) Sam: If three-and-a-half-sevenths is one-half, then three-sevenths is less than a half.

(14) Dana: Oh! And five-eighths is more than a half, so five-eighths has to be larger! *[The students record their answer.]* Next one. Four-fifths and six-sevenths…*[Dana tries to plot 4/5 on one number line and 6/7 on another just below. The two points look very close.]* These are too close to tell. I don't know if I drew the two points in exactly the right place, so we can't trust my drawing to decide which fraction is bigger. Drawing is just not good enough!

(15) Sam: And they are both bigger than a half and less than one, so we can't use that to help us decide.

(16) Anita: Hmmm, they are both really *close* to one though. Maybe we could use that.

(17) Dana: *[referring to the number lines]* Oh! Right! Sixth-sevenths is one-seventh away from one, and four-fifths is one-fifth away from one.

(18) Anita: Ah! We know one-fifth is bigger than one-seventh, so…

(19) Sam: Oooh, we have to think carefully here. One-fifth is bigger; so, four-fifths is *further* from one than six-sevenths is.

(20) Dana: Yes! So, six-sevenths is bigger! *[They record their answer.]* Now what about four-fifths and five-sevenths?

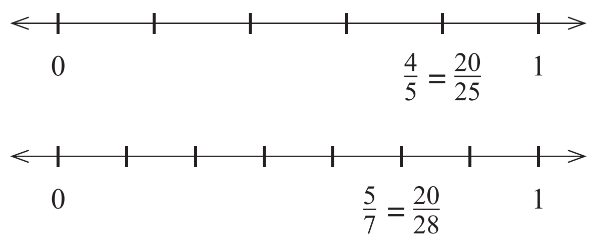
(21) Sam: Well, the numerators and denominators are different, and they are both bigger than one-half. So, compare to one?

(22) Dana: Ok, four-fifths is one-fifth away from one and five-sevenths is two-sevenths away from one. But now we have to compare one-fifth and two-sevenths. This isn't really working… What else can we do?

(23) Dana: This would be easier if either the numerators or the denominators were the same.

(24) Anita: *[Anita writes while talking.]* Well, can we make the numerators the same? If we multiply 4/5 by 5/5, which is the same as 1, we get 20/25. And then multiply 5/7 by 4/4 so that the numerators will be the same… then we get 20/28. Now we just need to compare 20/25 and 20/28. Twenty-fifths are larger than twenty-eights, so…

(25) Dana: So, 20/25 is bigger… and that means four-fifths is bigger!



(26) Sam: There are so many different ways to do it! How can we decide when each method will be the least work?

Teacher Reflection Questions

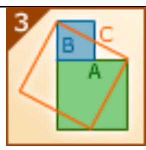
Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?
2. How is the use of number lines supporting these students?
3. In the last line of the Student Dialogue, Sam wonders how they can describe the cases in which each of their methods is optimal. What are the cases where each of the students' strategies work?
4. Beyond the cases examined in the Student Dialogue and in Question 3, what other fraction pairs might you give to students to compare to help them generalize these ways of thinking about fractions?
5. What strategies have you seen your own students use when comparing fractions?
6. What contexts might you attach to a fractions comparison task for students such as the one the students explore in this Student Dialogue, and why?
7. Explain exactly *when* and *why* it works to compare two fractions by looking at how far each of them is from 1.

Mathematical Overview

Suggested Use
The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	Evidence
 Reason abstractly and quantitatively.	The students are engaging in this practice when reasoning about the relationship between the quantities in each fraction pair they are comparing. They also "consider the units," particularly in line 16, when Anita says, "they are both really close to one." Further, they pay attention to "the meaning of the quantities, not just how to compute them," as when Anita says in line 11, "So three-and-a-half-sevenths is one half."
 Construct a viable argument and critique the reasoning of others.	The progression of insights expressed in the interactions from lines 16 through 20 builds an argument that 6/7 is a larger number than 4/5. It is, in the words from the SMP document, a "logical progression of statements."
 Use appropriate tools strategically.	The students make use of a tool appropriate for comparing fractions, namely, a number line. They appear to do so strategically, as well, particularly when, in comparing 5/8 and 3/7, Anita says (line 8), "Maybe we could just *think* about where they are on the number line." And that propels Dana to offer, "Well, five-eighths is a little more than a half," and that sets in motion the strategy of using 1/2 as a benchmark for comparison.
 Look for and make use of structure.	The students are considering the structure of the fractions and their placement on the number line as they consider methods for comparison of fractions. Throughout the Student Dialogue, they think about the role of the numerator and the denominator in understanding the magnitude of the fraction, and of the unit fractions that compose the fractions.

Comparing Fractions

Commentary on the Mathematics

It is important that middle graders develop the capacity to think of fractions as *numbers* that have magnitudes, positions on the number line, and distances from other numbers. To understand a fraction as a number means knowing that symbols like 3/5, 5/2, 3 ½ all represent magnitudes, just as 4, 12, and 135 do. Cognitive research has shown a close relationship between understandings of fraction magnitudes and fractions arithmetic. So, for example, when given the task of adding 4/5 and 2/9, students who understand that 4/5 and 2/9 are numbers with magnitude are unlikely to fall into the common error of taking 4+2 and 5+9 and concluding that the sum of the two fractions is 6/14, since they know that 6/14 is smaller in magnitude than 4/5. Students also need to understand that a number with a given magnitude (and located in a particular position on the number line) can be represented in many different ways—e.g., the number that 3/5 represents has infinitely many other notations (48/80, 0.6, 6.0×10^{-1}) as does 128.

While fraction *notation* can be interpreted as indicating number (3/5 is smaller than 2/3), it can also be interpreted as a portion of a group (3 parts of 5), as a ratio (in a 3:5 ratio), and as division (3 divided by 5).

In this Student Dialogue, by testing their ideas on the number line (as Dana does in line 2 and Sam suggests in line 7), the students are reasoning about fractions as numbers with magnitude. Note that, in line 2, Dana's language suggests thinking of fractions as division of two numbers, but the action of measuring along the number line implies that Dana also is thinking in terms of magnitude. Similarly, in line 11, when Anita says "So three-and-a-half sevenths is one half," she *could* be thinking about parts of groups—that 3½ is half of 7—but Sam's statement puts it in the context of placement on the number line, a magnitude.

In this Student Dialogue, students also use (without having a formal name for it) another important mathematical idea: *complements*—related to the word *complete*—the distance between what you have and a target quantity that is important for some reason. In high school geometry, we speak of "complementary angles" as angles that, together, complete 90°. In the Student Dialogue, students (line 17) think of how, starting from 4/5, they need 1/5 to complete a 1.

Evidence of the Content Standards

Throughout the student interactions, Content Standard 6.NS.C.6 is evident, as the students illustrate they understand rational numbers as points on the number line. Also evident throughout the interactions is Content Standard 4.NF.A.2, as the students compare fractions by reasoning about fractions with the same numerator by using benchmarks and by using common denominators. In line 2, Content Standard 5.NF.B.3 appears through Dana's reasoning that "one-fourth has to be bigger because it cuts 1 into only 4 pieces, and one-fifth cuts it into more pieces." Dana's use of the language of division suggests knowledge that a fraction represents the division of two numbers. Note: While standards from each of grades 4, 5, and 6 are relevant to what the students do with the task, teachers in the lower grades of this range may find that their students can only do some, not all, of the parts of the task correctly, or that they reason about them differently than older students might.

Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. In lines 11–13 of the student dialogue, Anita, Sam, and Dana figured out whether 3/7 is bigger or smaller than 1/2 in order to compare it to 5/8, which they knew was bigger than 1/2. For the other fractions used in this task (3/4, 3/5, 6/7, 4/5, 5/7), which are bigger than 1/2 and which are smaller than 1/2 and how do you know?

2. In line 11, Anita writes $\frac{3\frac{1}{2}}{7}$ and says "I've never seen a fraction like that before." Show *why* it makes complete sense that this funny-looking fraction equals 1/2. Invent five more fractions that equal 1/2, and make at least two of them such that you've "never seen a fraction like that before."

3. Invent your own pair of fractions that you can compare using the strategy Anita, Sam, and Dana used in lines 16–20? Show exactly how to compare your fractions using that strategy.

4. Now write your own pair of fractions that you can compare using the strategy Anita, Sam, and Dana used in lines 11–13? Show exactly how to compare your fractions using that strategy.

5. Invent eight fractions that all equal 1/3, and write them in the middle column of the table. Then put eight fractions in each of the other columns.

$< \frac{1}{3}$ (but positive)	$= \frac{1}{3}$	$> \frac{1}{3}$ (but less than 1)

6. **Challenge:** Use your thinking from Problem 5 to see if you can compare 4/9 and 11/30 without finding a common denominator.

7. Dana says in line 2 that 1/4 is bigger than 1/5. Explain, in your own words, her reasoning.

Comparing Fractions

Related Mathematics Tasks

1. Which fraction is larger: 5/8 or 4/7? How do you know?

2. Which is larger: 5/3 or 9/5? How do you know?

3. List the following set of fractions in order from largest to smallest: 5/17, 1/3, 6/11, 5/19, 3/8, 7/9, 4/9, 7/10. How do you know you have them in the right order?

Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

1. What evidence do you see of students in the dialogue engaging in the Standards for Mathematical Practice?

   Refer to the Mathematical Overview for notes related to this question.

2. How is the use of number lines supporting these students?

   Number lines support students in understanding fractions as a measure of quantity. The students are able to compare the fractions during this Student Dialogue by attending to their magnitudes and their relationship to each other. These students come up with a variety of strategies for comparing the fractions that depend on an understanding of how each fraction is composed of unit fractions. Additional resources for thinking about the role of number lines in students' fraction understanding include:

   - Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Number and Operations—Fractions, 3-5*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Retrieved from http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf
   - Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6–8, The Number System.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Retrieved from http://commoncoretools.me/wp-content/uploads/2013/07/ccssm_progression_NS+Number_2013-07-09.pdf
   - Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., …, & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide* (NCEE #2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/pdf/practice_guides/fractions_pg_093010.pdf

3. In the last line of the Student Dialogue, Sam wonders how they can describe the cases in which each of their methods is optimal. What are the cases where each of the students' strategies work?

   The students use four different strategies during the Student Dialogue. The first three are based on the idea of "distance" from some reference point. When the numerators are the same, only the denominators matter. They think about which unit fraction is larger by using its denominator to subdivide the interval from 0 to 1 on the number line and compare the size of the resulting steps. A given number of larger steps takes us farther from 0—that is, to a larger number—than the same number of smaller steps. This strategy obviously also works if the numerator of the fraction that represents "smaller steps" is less than the numerator of the other fraction.

To compare the next pair of fractions, 3/7 and 5/8, the students didn't think so much about distance as about whether the fractions were above or below a common "benchmark"—in this case, 1/2. They knew one fraction was less than 1/2 and the other was more than ½, and that solved the problem. This strategy works whenever the fractions being compared are on opposite sides (on a number line) of the benchmark.

The students' third strategy, comparing the two fractions' distances from 1, is equivalent to their first strategy, comparing their distances from 0. They realize 4/5 is 1/5 less than 1 and 6/7 is 1/7 less than 1. Since the numerators of the resulting fractions are the same, the larger denominator identifies a smaller fraction which means 6/7 is closer to 1.

For the last pair of fractions, students found a common numerator. This method, just like finding a common denominator, will work with any pair of fractions and was the easiest to apply for the last pair of fractions, 4/5 and 5/7.

Why bother with many methods? Though one can construct cases in which finding a common numerator or denominator becomes awkward, and one of these other methods can come to the rescue, such cases are purely artificial. The real reason is that the understanding about fractions that one gains from the other methods is, by itself, worth the extra time spent learning to think in those ways.

4. Beyond the cases examined in the Student Dialogue and in Question 3, what other fraction pairs might you give to students to compare to help them generalize these ways of thinking about fractions?

   Of course, more practice of the strategies used by the students in the Student Dialogue helps students become adept with these ways of thinking about fractions before generalizing, so invent some more fraction pairs in which the numerators are the same (e.g., 5/8 and 5/9, 3/16 and 3/20) to exercise the first strategy, and more fraction pairs where the students' third strategy (distance from 1) might work (e.g., 8/9 and 7/8 or 6/11 and 4/9). In addition to giving students more practice comparing to 1/2 (second strategy in the Student Dialogue), you might ask students to invent similar problems in which the "benchmark" to compare fractions to is 1/3. That can help them generalize the idea that they used with 1/2. They can also generalize strategies 1 and 3 by thinking about distance from 1/2. For example, which is greater, 3/8 or 5/12? They are both less than ½, but one of them is 1/8 less and one is 1/12 less, so the second one is larger. A similar (but harder) example is comparing 3/7 to 4/9—they are, respectively, a half-seventh and a half-ninth (or 1/14 and 1/18) below 1/2, so the closer one is larger. Other examples might push students to consider fractions greater than 1 (e.g., 5/3 or 8/5) or to consider negative numbers (e.g., –5/8 and –5/9).

5. What strategies have you seen your own students use when comparing fractions?

   Discuss with colleagues to share ideas.

6. What contexts might you attach to a fractions comparison task for students such as the one the students explore in this Student Dialogue, and why?

   Using a context for a fractions task can add relevancy for students and make the task more concrete. Furthermore, a context could add additional opportunities for students to possibly engage in MP 2: Reason abstractly and quantitatively, as they may engage in contextualizing and decontextualizing as they consider the meaning of the fractions under consideration. A wide variety of contexts could apply to the task in this Student Dialogue. For example, the task could be asking students to compare the stated serving sizes for different brands of ice cream (e.g., Which is a bigger serving size, 3/4 cup or 3/5 cup?). Students could also be asked to come up with a context that would fit with such a task.

7. Explain exactly *when* and *why* it works to compare two fractions by looking at how far each of them is from 1.

   Why it works follows from standard 6.NS.C.6 (cited in this Illustration) and MP 7. Conceiving of fractions as numbers means you can take advantage of the structure of the real numbers. In this case, if a number M is smaller than a number N, which in turn is smaller than a number P—so, $M < N < P$—then the distance between N and P is smaller than the distance between M and P. Conversely, if we know that the distance between N and P is smaller than the distance between M and P, then we can conclude that $M < N$. So, for example, if two fractions are each a unit fraction away from 1, then it is easy to compare the distance from each fraction to 1. For 4/5 and 6/7, 4/5 is 1/5 from 1 and 6/7 is 1/7 from 1… the distance from 6/7 to 1 is smaller, so 6/7 is the larger fraction.

   Similarly, if the distance from both fractions to 1 have the same numerator, even if that distance is not a unit fraction, it is easy enough to compare the size of those distances. So, if comparing 3/5 and 5/7, one is 2/5 away from 1 and the other is 2/7 away from 1. Sevenths are smaller than fifths, so 5/7 is closer to 1, and is the greater number. If the numerators are not the same when finding the distance to 1 for each fraction, then it is still possible to determine which fraction is greater if the distance-from-1 fraction with the greater denominator has the smaller numerator. For example, when comparing 3/5 and 6/7, 3/5 is 2/5 from 1 and 6/7 is 1/7 from 1. Fifths are greater than sevenths, and there are more of them (2 rather than 1), so it is clear that 2/5 is greater than 1/7, and that 6/7 is closer to 1. Sometimes, the strategy may not be the best way to compare two fractions. For example, if comparing 4/5 and 5/7, which are 1/5 and 2/7 away from 1, respectively, fifths are larger than sevenths, but there are more of the smaller sevenths, so it is not as immediately apparent which number is greater using this method.)

Comparing Fractions

Possible Responses to Student Discussion Questions

1. In lines 11–13 of the student dialogue, Anita, Sam, and Dana figured out whether 3/7 is bigger or smaller than 1/2 in order to compare it to 5/8, which they knew was bigger than 1/2. For the other fractions used in this task (3/4, 3/5, 6/7, 4/5, 5/7), which are bigger than 1/2 and which are smaller than 1/2 and how do you know?

   All of the other fractions in this task are bigger than 1/2. 3/4 is bigger than 2/4; 3/5 and 4/5 are both bigger than $\frac{2\frac{1}{2}}{5}$; and 5/7 and 6/7 are both bigger than $\frac{3\frac{1}{2}}{7}$.

2. In line 11, Anita writes $\frac{3\frac{1}{2}}{7}$ and says “I’ve never seen a fraction like that before.” Show *why* it makes complete sense that this funny-looking fraction equals 1/2. Invent five more fractions that equal 1/2, and make at least two of them such that you’ve “never seen a fraction like that before.”

   There are many possibilities, depending on what students know. Here are a few:
   $\frac{234}{468}$, $\frac{2\frac{1}{4}}{4\frac{1}{2}}$, $\frac{\frac{1}{2}}{1}$, $\frac{3\frac{1}{3}}{6\frac{2}{3}}$, $\frac{2.5}{5}$, $\frac{0.1}{0.2}$, $\frac{0.1}{\frac{2}{10}}$, and so on.

3. Invent your own pair of fractions that you can compare using the strategy Anita, Sam, and Dana used in lines 16–20? Show exactly how to compare your fractions using that strategy.

   The complements to their fractions—that is, the fractions that tell the distance from their initial fractions and 1—must have the same numerators. Listen for students to explain that when both fractions are less than 1, the one that is closer to 1 is the larger. This involves comparing denominators and realizing that when the numerators are the same, the fraction with the smaller denominator is the larger fraction. The fraction with the larger complement is farther from 1 and the fraction with the smaller complement is closer to 1.

4. Now write your own pair of fractions that you can compare using the strategy Anita, Sam, and Dana used in lines 11–13? Show exactly how to compare your fractions using that strategy.

   Listen for students to compare to 1/2 as a benchmark fraction (e.g., because one is bigger and one smaller than 1/2).

5. Invent eight fractions that all equal 1/3, and write them in the middle column of the table. Then put eight fractions in each of the other columns.

$< \frac{1}{3}$ (but positive)	$= \frac{1}{3}$	$> \frac{1}{3}$ (but less than 1)

Check student answers. Have students explain their thinking about how they know the fraction they are proposing is equal to, less than or greater than 1/3.

6. **Challenge:** Use your thinking from Problem 5 to see if you can compare 4/9 and 11/30 without finding a common denominator.

This is a real challenge. They are both greater than 1/3. We know because 3/9 and 10/30 are exactly 1/3. But the fractions we are comparing are more than 1/3 by different amounts. 4/9 is a *ninth* more, and 11/30 is only a *thirtieth* more, so 4/9 is greater than 11/30.

7. Dana says in line 2 that 1/4 is bigger than 1/5. Explain, in your own words, her reasoning.

Five *is* bigger than four, but that's why 1/4 is bigger. As Dana explained, the denominators say how many equal-sized pieces the distance from 0 to 1 must be broken into. If we break it into more pieces (5 rather than 4), then those pieces will be smaller. So, one-fifth is a smaller piece than one-fourth.

Possible Responses to Related Mathematics Tasks

1. Which fraction is larger: 5/8 or 4/7? How do you know?

5/8 is 3/8 away from 1 and 4/7 is 3/7 away from 1. Sevenths are bigger than eighths, so 4/7 is farther from 1, meaning that 5/8 is the larger number.

2. Which is larger: 5/3 or 9/5? How do you know?

9/5 is larger than 5/3. 5/3 is 1/3 away from 2 while 9/5 is 1/5 away from 2. 1/3 is a longer distance from 2 (breaking a unit into three pieces) than 1/5 is (breaking a unit into five pieces), so 9/5 is closer to 2 than 5/3, and is the larger number.

3. List the following set of fractions in order from largest to smallest: 5/17, 1/3, 6/11, 5/19, 3/8, 7/9, 4/9, 7/10. How do you know you have them in the right order?

   7/10 (largest), 7/9, 6/11, 4/9, 3/8, 1/3, 5/17, 5/19 (smallest)

   There are many methods for comparing the various fractions and ordering them. For example, group all the fractions that are less than half and all the fractions that are greater than half by comparing each to half.

   Three fractions are larger than 1/2 (7/9, 7/10, and 6/11) so those are the three largest. 7/9 is larger than 7/10 because ninths are bigger than tenths. Using common denominators, you can determine that 6/11 is smaller than 7/10. So, the largest three in order are 7/9, 7/10, then 6/11.

   The remaining fractions could be compared to 1/3. 3/9 is one-third, so 4/9 is greater than one-third and so is 3/8 because eighths are bigger than ninths. 5/19 and 5/17 are both smaller than one-third because 5/15 equals one-third and nineteenths and seventeenths are both smaller than fifteenths.

   So, now we know that 5/17 and 5/19 are the two smallest (smaller than 1/3) and 5/19 is the very smallest because nineteenths are smaller than seventeenths. 1/3 is then third smallest, and we just need to sort out the order for 3/8 and 4/9 (the fractions larger than 1/3 but smaller than 1/2). 3/8 is 5/8 away from 1 and 4/9 is 5/9 away from 1. Ninths are smaller than eighths, so 4/9 is closer to 1, and is therefore larger than 3/8.