Making Sense of Unusual Results

About Illustrations: Illustrations of the Standards for Mathematical Practice (SMP) consist of several pieces, including a mathematics task, student dialogue, mathematical overview, teacher reflection questions, and student materials. While the primary use of Illustrations is for teacher learning about the SMP, some components may be used in the classroom with students. These include the mathematics task, student dialogue, and student materials. For additional Illustrations or to learn about a professional development curriculum centered around the use of Illustrations, please visit mathpractices.edc.org.

About the *Making Sense of Unusual Results* Illustration: This Illustration's student dialogue shows the conversation among three students who are trying to solve an equation, and when the answer they first get (6 = 3) does not make sense, they reason about what might have happened. During their discussion students realize that that $x = 0$ is a solution which lead to their unusual result when they accidently divided by zero. Ultimately, students develop a different approach to solve the equation and agree on a check whenever they divide by a variable.

Highlighted Standard(s) for Mathematical Practice (MP)
MP 1: Make sense of problems and persevere in solving them.
MP 7: Look for and make use of structure.
MP 8: Look for and express regularity in repeated reasoning.

Target Grade Level: Grades 6–8

Target Content Domain: The Number System, Expressions & Equations

Highlighted Standard(s) for Mathematical Content

6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
7.NS.A.2b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.
8.EE.C.7	Solve linear equations in one variable.

Math Topic Keywords: distribution, rewriting expressions, solving equations, finding a missing solution, dividing by zero, checking answers

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Mathematics Task

Suggested Use
This mathematics task is intended to encourage the use of mathematical practices. Keep track of ideas, strategies, and questions that you pursue as you work on the task. Also reflect on the mathematical practices you used when working on this task.

Solve the equation $6(x + 3) = 3(x + 10) - 12$.

Student Dialogue

Suggested Use
The dialogue shows one way that students might engage in the mathematical practices as they work on the mathematics task from this Illustration. Read the student dialogue and identify the ideas, strategies, and questions that the students pursue as they work on the task.

Students have been learning to solve equations with one variable. They are familiar with standard rules for solving and algebraic manipulation.

(1) Sam: We are trying to solve this problem, so we need all the numbers that x can be that make the equation true. Don't we have to get the variable x by itself?

(2) Dana: Yeah, but we have to simplify both sides of the equation first.

(3) Sam: OK… so we get $3x + 30 - 12$. That's $3x + 18$.

(4) Anita: And on the left is $6x + 18$.
[Anita writes the equation $6x + 18 = 3x + 18$ *on the board.]*

(5) Sam: Great, and we can subtract 18 from both sides to get $6x = 3x$.

(6) Dana: And then we can divide by x to get $6 = 3$… Hang on. That doesn't make any sense. Six can't equal three!

(7) Anita: Of course. We want to keep x in the equation so we can figure out what it is equal to. Instead we got rid of it. Oh! I know, x can be zero! Look: 6 times 0 equals 3 times 0. So $x = 0$ should work in the original equation!

(8) Dana: Ok, for the left side, we get $6(0 + 3)$ which is $6 \cdot 3$ and that's 18.

(9) Sam: And for the right, we get $3(0 + 10) - 12$. That gives us $30 - 12$, which is also 18. So, $x = 0$ really is a solution, but how come we didn't get that when we solved?

[The students write out their solving process and reflect for a few minutes.]

$$
\begin{gathered}
6(x + 3) = 3(x + 10) - 12 \\
6x + 18 = 3x + 30 - 12 \\
6x + 18 = 3x + 18 \\
-18 \quad -18 \\
6x = 3x \\
\div x \quad \div x \\
6 = 3
\end{gathered}
$$

(10) Anita: Well, like I said, we got rid of that x altogether. Oh, and also, now that I'm looking at it, we divided by x, and x is 0. You can't divide by zero.

(11) Dana: I know that dividing by *zero* makes no sense, but we divided by x.

(12) Michael: I guess if you divide by a variable, and that variable actually equals zero, then you kind of break things.

(13) Dana: Oh! Right! In our case, x *is* zero! OK, so we need to watch out for that.

(14) Anita: Well, really, we should have finished what Sam said first thing about getting all the x's into one place, or something like that.

(15) Sam: I said we have to get the variable x by itself.

(16) Anita: Exactly, so when we had $6x = 3x$, it would have made more sense to subtract $3x$ from each side instead of dividing by x.

[The students return to the $6x = 3x$ *step, and solve by subtracting* $3x$ *first.]*

$$6x = 3x$$
$$-3x \quad -3x$$
$$3x = 0$$
$$\div 3 \quad \div 3$$
$$x = 0$$

(17) Dana: Well, at least that gives the answer we already know is right!

(18) Sam: So, is dividing by x wrong?

(19) Anita: I think it would be fine if x were 5 or something. We just can't divide by zero. If x could be zero, then we cause problems when we divide by x.

[The students record their own work on their pages and look over some other problems.]

(20) Sam: Ok, what about a problem like $11(x + 5) = 7(x + 5)$? If we divide by $x + 5$, we get the same kind of thing: $11 = 7$, but $x = 0$ isn't a solution to this one.

(21) Dana: Well, we could just simplify again to get $11x + 55 = 7x + 35$ and then….

[Dana writes out her solving process.]

$$11(x + 5) = 7(x + 5)$$
$$11x + 55 = 7x + 35$$
$$-55 \quad -55$$
$$11x = 7x - 20$$
$$-7x \quad -7x$$
$$4x = -20$$
$$\div 4 \quad \div 4$$
$$x = -5$$

(22) Sam: So, $x = -5$ is the solution!

(23) Anita: Well, actually, we could have figured that out way back before you got $11 = 7$, Sam. Look…when you divided $11(x + 5) = 7(x + 5)$ by $(x + 5)$, you got $11 = 7$. But that weird result of $11 = 7$ is a clue that something has gone wrong. All we did is that one division, so that must be the cause of the problem, and the only division that's not logical is division by 0. So if $x + 5$ *is* equal to zero, then that would mean that $x = -5$.

(24) Sam: Ok. And, we could just test that solution in the original equation rather than writing out all those steps.

[Sam tests the solution x = –5 in the original equation.]

$$11(-5 + 5) = 7(-5 + 5)$$
$$11 \cdot 0 = 7 \cdot 0$$
$$0 = 0$$

(25) Dana: So if we are solving, and we divide by *anything* with a variable in it, then we have to keep track of the possibility that what we are dividing by is equal to zero.

Teacher Reflection Questions

Suggested Use
These teacher reflection questions are intended to prompt thinking about 1) the mathematical practices, 2) the mathematical content that relates to and extends the mathematics task in this Illustration, 3) student thinking, and 4) teaching practices. Reflect on each of the questions, referring to the student dialogue as needed. Please note that some of the mathematics extension tasks presented in these teacher reflection questions are meant for teacher exploration, to prompt teacher engagement in the mathematical practices, and may not be appropriate for student use.

1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

2. What mistakes in algebraic manipulation are students likely to make, and how might you help to clarify their thinking?

3. Why is it that knowing $xy = 0$ implies that either $x = 0$, $y = 0$, or both? How would you discuss this implication with students? Does this implication hold with other number systems?

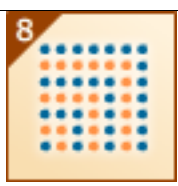
4. In this Student Dialogue, students ran into trouble when they divided by a variable that could be zero. What other operations could cause problems when solving equations?

5. After line 9, how might you intervene if the students did not come up with a reason for why they did not get $x = 0$ as a solution when they solved for x?

Mathematical Overview

Suggested Use
The mathematical overview provides a perspective on 1) how students in the dialogue engaged in the mathematical practices and 2) the mathematical content and its extensions. Read the mathematical overview and reflect on any questions or thoughts it provokes.

Commentary on the Student Thinking

Mathematical Practice	Evidence
 Make sense of problems and persevere in solving them.	In line 7, Anita suggests "So $x = 0$ should work in the original equation!" and then the students test whether it does. In line 24, Sam does the same with the solution $x = -5$. These examples are consistent with the part of the description of MP 1 that says "Mathematically proficient students check their answers to problems using a different method."
 Look for and make use of structure.	In lines 14–16, Anita and Sam discuss using an approach that avoids dividing by x. This exchange suggests that they "step back for an overview and shift perspective," as is described by MP 7. In line 20, Sam introduces an example to test the group's thinking about missing solutions: the problem $11(x + 5) = 7(x + 5)$. After the students work through their solution process, Anita, in line 23, suggests that maybe the steps they went through are not necessary and that looking at the form of the equation could lead them to see that $(x + 5) = 0$. This is, in the language of MP 7, seeing "complicated things, such as some algebraic expressions, as single objects."
 Look for and express regularity in repeated reasoning.	In line 25, Dana summarizes "So if we are solving, and we divide by *anything* with a variable in it, then we have to keep track of the possibility that what we are dividing by is equal to zero." Here, she is suggesting a general rule for them to remember in the solving process, and her comment is consistent with the MP 8 statement, "Mathematically proficient students… look both for general methods and for shortcuts." Dana's statement is an example of looking for a general rule or method, and Anita's thinking behind the statement in line 23 is an example of looking for shortcuts.

Commentary on the Mathematics

In beginning algebra, students learn to use the structure of algebraic expressions, thus employing MP 7 (for example, in line 16, "when we had $6x = 3x$, it would have made more sense to subtract $3x$ from each side instead of dividing by x"). The nature of the equation they are solving forces the students to make explicit their understanding of the structure of the real number system, namely, that while every real number has an additive inverse, not every real number has a multiplicative inverse (zero is the exception). In line 11, when Dana says, "I know that dividing by *zero* makes no sense, but we divided by x," she is giving voice to a common quandary students have in using division involving unknowns to move toward solution: they can end up in a nonsensical statement, like 6 = 3, or they can lose a solution, as in dividing by x to solve the equation $x^2 = x$. Here, dividing by x yields a correct solution $x = 1$, but another correct solution $x = 0$ is lost. In the end (line 25), Dana makes sense of the overall situation by saying, "So if we are solving, and we divide by *anything* with a variable in it, then we have to keep track of the possibility that what we are dividing by is equal to zero." Another way the students might have sought a way out of their quandary is to graph the two sides of the given equation as separate linear functions $y = 6(x + 3)$ and $y = 3(x + 10) - 12$ to see that they intersect at $x = 0$.

The zero product property of real numbers (when two numbers have a product of zero, at least one of the numbers must be zero) does not hold for all number systems, but it does hold for all systems *with multiplicative inverses* for all non-zero elements, commutativity, and associativity. (A proof for real numbers is shown in the Teacher Discussion Questions, but it applies equally well for any system with multiplicative inverses.) Additionally, some systems *without* multiplicative inverses for all non-zero elements still have the zero product property, such as the integers (if integers m and n have a product of 0, that is, $mn = 0$, then either $m = 0$, $n = 0$, or both, even though no integers have multiplicative inverses that are themselves integers) and polynomials (for example, $x(x - 1) = 0$ implies that either $x = 0$, $x = 1$, or both, even though neither inverse, x^{-1} nor $(x - 1)^{-1}$, is itself a polynomial). Examples of systems in which the zero product property does *not* hold are given in the Teacher Discussion Questions.

When Anita treats $(x + 5)$ as something they can compute with (line 23), she is illustrating the part of MP 7 that describes how proficient mathematics learners see "complicated things, such as some algebraic expressions, as single objects." Developing this capacity is an important step in algebraic learning. It is akin to students learning to switch between different "wholes" in solving rational number situations, as in "Maria gave $\frac{1}{4}$ of her candies to Carl and $\frac{1}{3}$ of the remaining candies to Alice," where it is necessary to switch from considering all of Maria's candies as the whole to a new whole consisting only of what remains after giving Carl some.

<u>Evidence of the Content Standards</u>
Content Standard 8.EE.C.7, "Solve linear equations in one variable," is evident throughout the Student Dialogue as it is the purpose of the task itself. Likewise, 7.NS.A.2b appears throughout the Student Dialogue as students grapple with the issue of dividing expressions that could be equal to zero. Content Standard 6.EE.B.5 appears in two ways: First, when the students confront the surprise that there seems to be no solution (line 6) and need to consider what went wrong in their algebraic manipulations, they are answering the question, "Which values from a specified set, if any, make the equation or inequality true?" (In this Student Dialogue, as is usually the case in middle school mathematics, the "specified set" of possible solutions is the set of real numbers.) Second, they twice, "[u]se substitution to determine whether a given number in a specified set makes an equation or inequality true."

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Student Materials

Suggested Use
Student discussion questions and related mathematics tasks are supplementary materials intended for optional classroom use with students. If you choose to use the mathematics task and student dialogue with your students, the discussion questions can stimulate student conversation and further exploration of the mathematics. Related mathematics tasks provide students an opportunity to engage in the mathematical practices as they connect to content that is similar to, or an extension of, that found in the mathematics task featured in the student dialogue.

Student Discussion Questions

1. What does it mean to solve an equation?

2. At the end of the Student Dialogue, Dana says, "if we are solving, and we divide by *anything* with a variable in it then we have to keep track of the possibility that what we are dividing by is equal to zero." What does Dana mean?

3. Why does Dana say "*anything* with a variable in it" (line 25)? Why don't we have to do the same check when we divide by something without a variable in it?

Related Mathematics Tasks

1. Imagine Sam and Anita are solving the equation $8(x - 4) = 6(x - 4) - 8$. How might you suggest they consider this problem?

2. Imagine Sam and Anita are *now* solving the equation $3(x + 3) = 3(x + 10) - 21$. They simplify it to $3x + 9 = 3x + 9$, then to $3x = 3x$, then to $3 = 3$. This time, there is no contradiction. Three *does* equal three. But they're not sure what to say about the solution. How might you help them think about this?

3. Imagine Anita and Dana are solving the problem $(2x - 1)(x + 5) = (x + 1)(x + 5)$. How might you suggest they consider this problem?

4. Now Sam and Dana are solving the equation $x^2 + 7x = 0$. They divide by x to get $x + 7 = 0$, and they conclude that $x = -7$ is the solution. Are they right? What would you say to them?

5. Create another problem that could end up with a missing solution (similar to how the $x = 0$ solution could get lost in Question #4), and share it with someone else. Then discuss how you each solved the other's problem.

Answer Key

Suggested Use
The possible responses provided to the teacher reflection questions are meant to be used as an additional perspective after teachers have thought about those questions themselves. The possible responses to the student discussion questions and related mathematics tasks are intended to help teachers prepare for using the student materials in the classroom.

Possible Responses to Teacher Reflection Questions

1. What evidence do you see of the students in the dialogue engaging in the Standards for Mathematical Practice?

   Refer to the Mathematical Overview for notes related to this question.

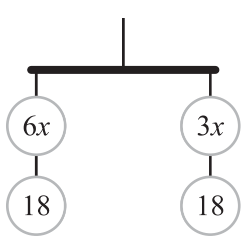
2. What mistakes in algebraic manipulation are students likely to make, and how might you help to clarify their thinking?

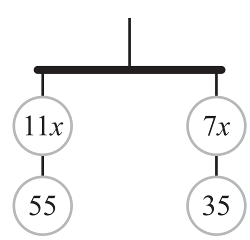
   Discuss with colleagues to share ideas.

   A common student error is to forget to distribute the multiplication, instead applying the multiplication to only one term inside the parentheses, concluding, for example, that $6(x + 3) = 6x + 3$. This could be just a slip, but may also indicate a misunderstanding of the process of multiplying polynomial expressions. This can be clarified through the use of an area model such as the one shown below.

	x	3
6	$6x$	18

   Students who have learned equation-solving methods as mechanical rules can make the common mistake of subtracting a term from one side and adding it to the other. This "move it over and change the sign" action is a generalization—but a wrong one—of the action-pattern that one sees in transformations like $x – 5 = 0$ to $x = 5$. The *logic* of that situation makes clear that we are not "moving things" but using arithmetic/algebraic operations, in this case, adding 5 to both sides. Focusing on the logic, not the procedures or patterns, can help. One approach is the algebraic "operate the same way on both sides" suggested above. It can also be helpful to use the metaphor of a balancing mobile as shown in the models below.





These models often make it clearer that while subtracting (or adding) the same amount to each side will maintain balance, moving something from one side to the other (subtracting from one side and adding to the other) will not. These models can assist students in building logic of the solving process and can also help them to reason about why x must be zero in the first case in the Student Dialogue and negative in the second case in the Student Dialogue—before even beginning the formal solving process.

3. Why is it that knowing $xy = 0$ implies that either $x = 0$, $y = 0$, or both? How would you discuss this implication with students? Does this implication hold with other number systems?

Discuss with colleagues to share ideas.

For any two real numbers x and y, if we know $xy = 0$, then either $x = 0$ or $x \neq 0$. And if $x \neq 0$, then we can divide both sides of the original equation by x, and we get $y = 0$. Of course, if $x = 0$, it could also be true that $y = 0$, so either $x = 0$, $y = 0$, or both.

It may be fruitful to pose the initial question (Why is it that knowing $xy = 0$ implies that either $x = 0$, $y = 0$, or both?) to students and allow them time to consider and discuss. After students have had a few minutes to consider, it may be helpful to prompt with, "Let's consider x, though we could just as well consider y. It must be true that either $x = 0$ or $x \neq 0$. And if $x \neq 0$, then what else can we say?" If students have read through the Student Dialogue or have been working with similar ideas in class, someone may recognize that because they are assuming that $x \neq 0$, they can divide both sides by x to show that if $x \neq 0$ then $y = 0$. Encourage students to work toward the kind of logic used in the simple proof above.

This implication does not hold for all number systems. For instance, in clock arithmetic (that is, arithmetic modulo 12), $3 \cdot 4 = 0$ because $12 = 0 \pmod{12}$. When the hour hand on a clock moves from 12 through 4 periods of 3 hours each, it returns to its original starting place at 12 (which is congruent to 0). In this case, $3 \cdot 4 = 0$, but neither $3 = 0$ nor $4 = 0$. Likewise, the two non-zero matrices $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ have a product of zero:

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 0 & 1 \cdot 0 \\ 0 \cdot 1 & 0 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

. However in the integers, rational numbers, real numbers, and complex numbers, the zero product property holds. All systems with multiplicative inverses for all non-zero elements, commutativity, and associativity have the zero product property, and even some systems without multiplicative inverses (such as integers and polynomials), have the zero product property. The zero product property of integers and polynomials is described in the Mathematical Overview.

4. In this Student Dialogue, students ran into trouble when they divided by a variable that could be zero. What other operations could cause problems when solving equations?

   If students forget that all non-negative real numbers have two square roots, this can cause a solution to be lost. Consider solving $x^2 = 25$. Both $x = 5$ and $x = -5$ are solutions to this equation, but if someone only identifies the principal root ($x = 5$), then the $x = -5$ solution is missed.

   Several other problems involving division where students may miss a solution are included in the Related Mathematics Tasks.

5. After line 9, how might you intervene if the students did not come up with a reason for why they did not get $x = 0$ as a solution when they solved for x?

   If the students did not have the "A-ha" moment that they did in line 10, one option is to question them about when the contradiction happened. For example, it may make sense to say to students "You're absolutely right! This makes no sense! So, what, if anything, went wrong? Was the contradiction there from the start? Or can you find a step in which it could have crept in?" Nothing might have been done wrong; after all, the equation $4x + 6 = 4x + 3$ also generates $6 = 3$ without any error on the part of the students. The statement $4x + 6 = 4x +3$ is simply false from the start. Maybe that's true of ours, too. But the question of what might have gone wrong and why they didn't get what they expected is worth students asking. If that's not enough, you could give them equations that they can solve that involve combining all the x's, and then go back to the one in the Student Dialogue. Other good questions to ask include "Where'd the x go?! Don't you need it to stick around if you're going to figure out what number it is?!"

Possible Responses to Student Discussion Questions

1. What does it mean to solve an equation?

   As Sam points out in line 1, solving means finding, "all the numbers that x can be that make the equation true." While students may share (and certainly will benefit from) the procedural understanding that Sam indicates next, namely that, "we have to get the variable x by itself," all students should be able to identify that the solutions to an equation are the values that can be substituted in for the corresponding variables and will make the equation a true statement. Likewise, students should be aware that if a non-solution is plugged into an equation, then simplifying that equation will result in a false statement. This understanding leads directly to a process for verifying a solution found: plugging it in to check if it produces a true statement.

2. At the end of the Student Dialogue, Dana says, "if we are solving, and we divide by *anything* with a variable in it then we have to keep track of the possibility that what we are dividing by is equal to zero." What does Dana mean?

   Dana's statement sums up the ideas of the Student Dialogue. Use this question and the next as an opportunity for students to solidify their understanding of this Student Dialogue. Listen for students who understand why it's important to pay attention to what they divide by when solving (because we cannot divide by zero) and what can go wrong (if we do divide by something that does equal zero for some solutions, then the results will be illogical).

3. Why does Dana say "*anything* with a variable in it" (line 25)? Why don't we have to do the same check when we divide by something without a variable in it?

   This is an opportunity for students to dig into the ideas of this Student Dialogue more deeply. Listen for students who identify that if there is no variable in the divisor, then we can assess whether the value of the divisor is zero; otherwise, we do not know and risk generating illogical results. Listen to see if students understand that a variable by itself or an expression with a variable in it could both be equal to zero and, therefore, could present problems when used as a divisor.

Possible Responses to Related Mathematics Tasks

1. Imagine Sam and Anita are solving the equation $8(x - 4) = 6(x - 4) - 8$. How might you suggest they consider this problem?

   Simplifying and isolating x will certainly work, as long as students don't make the same mistake as in the Student Dialogue: dividing by x and neglecting to test whether $x = 0$ is a solution. Students may avoid this either by noticing that they have divided by an expression with a variable in it and testing $x = 0$ in the original equation or by subtracting $6x$ from each side to get $2x = 0$, thereby avoiding division by an expression with a variable entirely.

   Others may use structure, as described in MP 7, and subtract $6(x - 4)$ from both sides to get $2(x - 4) = -8$, which then means that $(x - 4) = -4$, and solve from there. Much less computational work is involved, because lots of the complexity is removed right at the start.

   Have students share their solving methods, and consider presenting one or more of the methods described above—especially if students did not generate more than one to discuss. Ask students to compare and contrast the methods presented: do these methods all make logical sense? Or are they risky? If so, at what step and why? Which solving methods require more steps? Why might you choose one method over another?

2. Imagine Sam and Anita are *now* solving the equation $3(x + 3) = 3(x + 10) - 21$. They simplify it to $3x + 9 = 3x + 9$, then to $3x = 3x$, then to $3 = 3$. This time, there is no contradiction. Three *does* equal three. But they're not sure what to say about the solution. How might you help them think about this?

   The last step, dividing by x, tossed out the very thing the students were trying to solve for (similar to what happened to the students early in the Student Dialogue, although in that case they ended up with a nonsensical answer of $6 = 3$). If they had divided by 3 in this case, they might not feel much better about the result: $x = x$. And, for that matter, if they looked at $3x = 3x$, and subtracted $3x$ from both sides, they'd end up with $0 = 0$, which is not more satisfying than $3 = 3$. So, what's the solution? Well, $x = x$, and $3 = 3$ and $0 = 0$ are all true, no matter what x is, so that is the solution! The x can be *any number*.

3. Imagine Anita and Dana are solving the problem $(2x - 1)(x + 5) = (x + 1)(x + 5)$. How might you suggest they consider this problem?

   This problem gives students the opportunity to work with an equation that has two solutions. Students may try a variety of solving methods: graphing the two sides of the equation as separate quadratic functions (perhaps using a graphing calculator or software to assist) and finding their intersection points; distributing and using the quadratic formula; or dividing by $x + 5$, solving, and then going back to check if $x + 5 = 0$ leads to a solution.

   Similar to Related Mathematical Task 1, you may wish to have students share several different solving methods (and perhaps present one yourself), discuss each one, and then compare and contrast the different methods.

4. Now Sam and Dana are solving the equation $x^2 + 7x = 0$. They divide by x to get $x + 7 = 0$, and they conclude that $x = -7$ is the solution. Are they right? What would you say to them?

   This is an opportunity for students to extend the same kind of reasoning to a different problem. By dividing by x, Sam and Dana have lost track of the solution $x = 0$. Listen for students checking that -7 is a solution (it is) and identifying that other solutions may have been lost when Sam and Dana divide by an expression with a variable in it. You may wish to refer back to the ideas in the Student Dialogue, in particular to line 25, Dana's observation that "if we are solving, and we divide by *anything* with a variable in it, then we have to keep track of the possibility that what we are dividing by is equal to zero." The students have divided by x; so they should check what happens when $x = 0$. In this case, $x = 0$ is a solution; so the two sides will be equivalent when 0 is substituted in for x: $x^2 + 7x = 0$ becomes $(0)^2 + 7(0) = 0$ which becomes $0 + 0 = 0$, which is a true statement.

5. Create another problem that could end up with a missing solution (similar to how the $x = 0$ solution could get lost in Question #4), and share it with someone else. Then discuss how you each solved the other's problem.

    Creating mathematical problems allows students to become producers rather than just consumers of mathematics—a powerful tool for engagement as it encourages students to develop mathematics for themselves as needed to solve problems. In addition, creating problems provides students the opportunity to see mathematics from the other side. For example here, to create a problem that could have a missing solution, students have to think about the contexts in which a missing solution could arise; they have to notice that this occurs when the same variable expression is a factor of both sides of the equation—a fact they will understand more deeply by discovering it in the process of creating a problem than they would if they were simply told.